What Every Engineer Should Know about Structures

Part C- Axial Strength of Materials

by

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Course 261

4 PDH (4 Hours)

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This is a continuation of a series of courses in the area of study of physics called engineering mechanics. This series of courses is called **Strength of Materials.** And this next course of the series is called **What Every Engineer Should Know About Structures - Part C - Axial Strength of Materials.**

The first courses of the series, called **Statics**, focused on solving problems related to the exterior (or externally) applied loads on a stationary body - a body at rest. It is expected that you have a good background in the study of Statics. If you are not familiar with statics, consider taking the two SunCam courses titled **What Every Engineer Should Know About Structures - Part A - Statics Fundamentals** and **What Every Engineer Should Know About Structures - Part B - Statics Applications**.

The study of **Strength of Materials** takes the next step and focuses on solving problems dealing with the stresses within those members of a stationary body (beams, columns, cables, etc.). This series will provide the tools for solving some of the most common structural design and analysis problems. The focus will be on presenting simplified methods of solving problems.

This course includes:

- stress and strain in a member, and their relationship, including material properties such as Hooke's Law and modulus of elasticity;
- axial loads in tension and compression, including deformation;
- shear stresses, shear modulus of elasticity, single and double shear, and punching shear;
- design stresses and factors of safety;
- temperature deformation and thermal stresses, and;
- cross sectional properties of structural members including determining the centroid of a cross section.

The next course in this series, **What Every Engineer Should Know About Structures - Part D – Bending Strength of Materials**, focuses on moments of inertia, torsional stresses and deformations, bending stresses including shear and moment diagrams, and deflection of beams.

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WHAT IS STRENGTH OF MATERIALS

Strength of Materials is part of the division of physics known as engineering mechanics. The study of **Strength of Materials** considers the internal forces in structural elements to determine:

1. the stress and deformation within the member (stress analysis);
2. the required size of the member to support the applied loads (design analysis); or
3. the load carrying capacity of a structural system or member (load analysis).

The first part of engineering mechanics, usually known as "statics", covers material related to maintaining equilibrium balance of a particular structural system with external loads (such as truss, frame, etc.) or a particular structural member (such as a beam, column, step ladder, etc.). External loads produce support reactions which maintain equilibrium of the structural system or member. The magnitude of the reactions are such that they create (or maintain) a zero sum of forces in the x-direction and the y- direction and the sum of the moments at any point equals zero. In statics, the structural system or member is assumed to be rigid - a rigid body. Assumed not to deform. And assumed to have unlimited strength

In real life, materials are not rigid and they do not have unlimited strength. Materials supporting external loads stretch, bend, and shorten; they get thinner or fatter; and they twist, and undergo other deformations - they are certainly not rigid. And, of course, all materials do have an upper limit on their strength and they fail when they reach that limit.

The applied external loads on a member are transmitted through the member to the supports (and vice versa) via internal forces within the member to maintain equilibrium. As this internal force transmission takes place, the structural system experiences stresses and deformations.

EQUILIBRIUM and SIGNIFICANT FIGURES

Equilibrium

All of Statics and Strength of Materials is based on equilibrium of a body at rest. In two dimensional systems, the sum of forces in the vertical direction always equals zero. The sum of forces in the horizontal direction always equals zero. And, the sum of the moments at any point always equals zero.

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Significant figures

Although most of us like our calculations to be precise, that is not always the case in real-life problems. Certainly we must know the procedures and the logic behind the calculations, but we must also know the assumptions made to make the problems solvable, and for the solutions to have enough accuracy to be meaningful.

Three significant figures for an "answer" is logical because of the following assumptions in the theory and the unknowns inherent in the problem:

- The magnitude of the applied loads are usually only approximate.
- The precise point of application of the load is difficult to determine with a high degree of accuracy.
- Point loads are not really point loads - they occur over some real area.
- "Frictionless" pins are not really frictionless.
- The structural materials are not perfectly homogeneous.
- Assumptions in the mathematics are often made to make the problem easily solvable.
- The conditions of service for the member are unknown for the entire future life of the member.
- Deflections - that effect the theory - are usually ignored in stress calculations.

For the above reasons, answers in Statics and Strength of Materials are usually given to only three significant figures. However the results of the intermediate calculations used to arrive at the answers are often carried to several significant figures.

Significant Figures, Rounding, and the Solution of a Statics Problem – A Quick Review is a zero credit course intended for those who might find themselves a bit rusty and would like a quick refresher. The information in the course is useful for application to solutions of structural problems.

This course is free and can be downloaded at:

http://www.suncam.com/authors/123Glon/statics.pdf

AXIAL LOADS

An axial load is a load on a member that has a line of action along the long axis of a member. Tension and compression loads are axial loads. The drawings below show axial loads.

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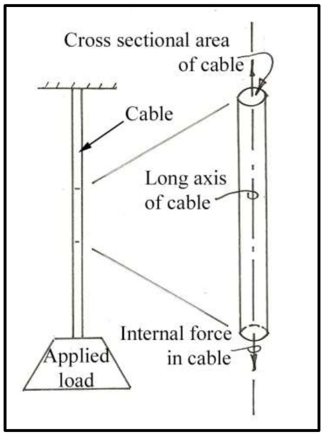
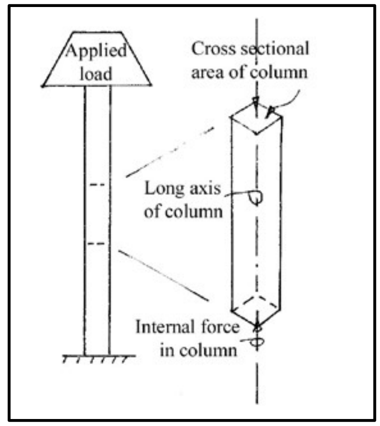


Axial Tensile Force
in a Cable



Axial Compressive Force
in a Column

STRESS REVEALED

Stress is a nifty concept. It reduces any force, acting on any area, to a common denominator - **unit stress**. Stress in a material is the internal resistance of a material to an externally applied load. In this basic form, all unit stresses are relatable to one another. Any member or material that encounters a stress of 100 pounds per square inch (psi) has a stress exactly twice as intense as any other member with a stress of 50 psi. The study of strength of materials depends on understanding the principles of stress.

Definition

The definition:

Unit stress - the magnitude of a force per unit area acting at some location.

The unit stress, or just plain stress, is denoted by the lower case Greek letter sigma (σ). The formula for unit stress is simple:

$$Unit\ Stress = Stress = \frac{Force}{Area}; \qquad \sigma = \frac{F}{A}$$

The definition of unit stress contains **one answer and three variables**. That's right, three variables.

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Answer

The **answer**, of course, is the value of unit stress. The most common units for unit stress in the study of strength of materials are pounds per square inch ($^{\#}/_{in^2}$ or psi); kips per square inch ($^{kips}/_{in^2}$ or ksi); and Newtons per square meter ($^{N}/_{m^2}$).

The three variables (in addition to the answer, σ) in the definition of unit stress are the **force** within the member, the **area** over which the force acts, and the **location** being considered.

Location

It is important to identify the location within (or along) the member where the unit stress will be determined (calculated). Stresses can vary along the length of a member. Frequently many points along the length of, say, a column, will have a different unit stress.

Area

The area used in the calculation of an axial unit stress is simply the cross-sectional area of the member at the location where the stress is wanted.

Force

The force used in the calculation of unit stress is an internal force within the member. The applied external loads (also called forces) on a member create internal reactions within a member to resist and transfer those external loads to supports, other members, etc.

The first step in finding the stress in a structural member at a particular point is determining the internal force in the element at that particular location. The internal forces on a member can be calculated using the principles of equilibrium - $\sum F_V = 0$; $\sum F_H = 0$; $\sum M = 0$.

The procedure for finding the internal force in a member is:

1) Draw a free body sketch of the entire structure
2) Isolate the location
    i) pass an imaginary "cutting plane" through the member at the desired location
    ii) draw a partial free-body diagram - either the left free-body or the right free-body, or the upper or lower free-body. Usually one of the free-bodies will be most convenient
3) Show the unknown internal forces that could possibly act at the cut section

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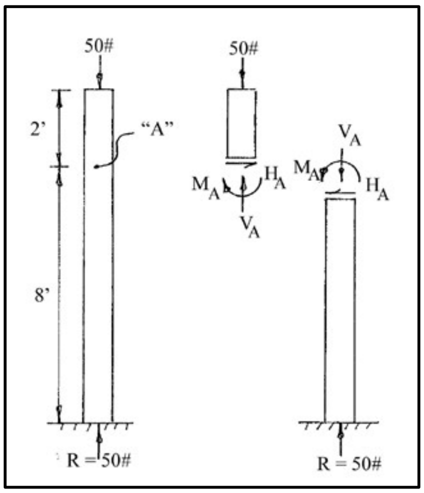
i) from statics, remember that there are three unknowns for equilibrium - a vertical force, a horizontal force, and a moment

4) Find the unknown force(s) using the equilibrium equations

Example Problem

Find the internal force at point "A" in the member shown below.

First, pass a plane through the member at the location desired (Point "A" in this case). Then list all the possible forces that could possibly act on that plane (H_A, V_A, and M_A). And, finally, using statics principles, determine the forces acting on Point "A" (or on the cross section at Point "A")



Calculations on upper free-body

$\Sigma F_H = 0 \Rightarrow H_A = 0$

$\Sigma F_V = 0 \Rightarrow V_A = 50\# \uparrow$

$\Sigma M @ A = 0 \Rightarrow M_A = 0$

Calculations on lower free-body

$\Sigma F_H = 0 \Rightarrow H_A = 0$

$\Sigma F_V = 0 \Rightarrow V_A = 50\# \downarrow$

$\Sigma M @ A = 0 \Rightarrow M_A = 0$

Notice in the problem above that the internal vertical force on the cross section is 50 pounds. And that it acts upward on the upper free-body diagram. And that it acts downward on the lower free-body diagram. This is consistent with the principles of equilibrium - each section of the free-body must be in equilibrium.

In the following example problems, we will only draw one section of the free-body diagram to solve the problems. Only one free-body is necessary since the forces are identical in magnitude (not in direction) on both free-bodies.

Now, let's work a couple of stress calculation problems ...

Example Problem

What is the stress in the member at point A in the drawing on the next page?

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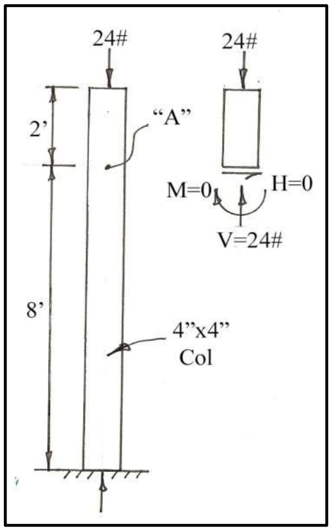
Solution Cut the member at point A and draw a free body diagram. In this case, let's use the top free-body diagram Show the possible forces that could act on that free-body diagram at Point "A". Then, solve for the forces (notice that there is only a vertical force at that point); calculate the area of the cross section; and finally solve for the stress in the member at Point "A".



Force at point A: $F = V = 24\# \uparrow$

Area of member at point A: $A = 4" \text{ x } 4" = 16 in^2$

Stress at Point A: $\sigma = \frac{F}{A} = \frac{24\ \#}{16\ in^2} = 1.5\ psi$

In this example problem the area of the column is constant along the length of the column. And the force in the column is also constant along the length of the column. Therefore, the stress in the member is constant along the length of the column. The point "A" could be anywhere along the length of the column and the stress would be the same.

Example Problem

What is the stress in the member at points A and B in the column shown on the next page?

In this example problem notice that the area of the column is constant for the entire length of the column. But in this case, the load is different at different points along the length of the column. Therefore, the stress in the member is different at points "A" and "B" along the length of the column.

Solution Cut the member at points "A" and "B" and draw the most convenient free-body diagrams; solve for the vertical forces (the only ones acting on the cross sections); and solve for the stress at each point - "A" and "B".

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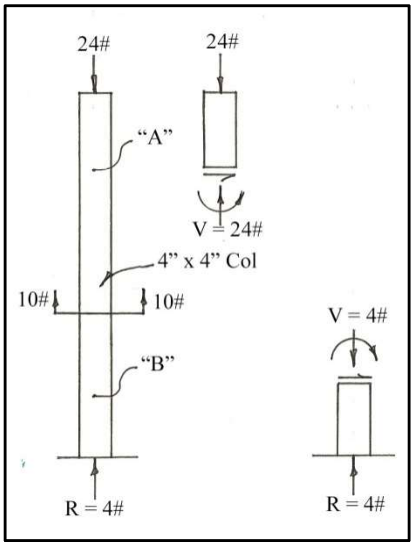


Cut the member at point "A" and draw a free body diagram.

Force at point A: $F = V = 24\# \uparrow$

Area of member at point A: $A = 4'' \text{ x } 4'' = 16 in^2$

Stress at Point A: $\sigma = \frac{F}{A} = \frac{24\ \#}{16\ in^2} = 1.5\ psi$

Cut the member at point B and draw a free body diagram.

Force at point B: $F = V = 4\# \downarrow$

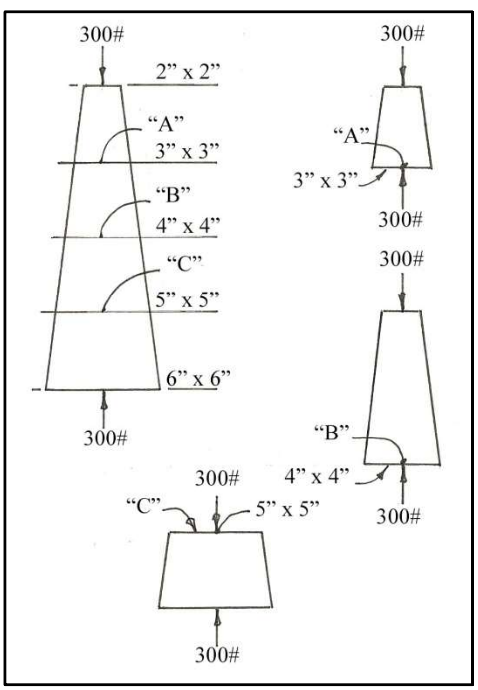
Area of member at point B: $A = 4'' \text{ x } 4'' = 16 in^2$

Stress at Point B: $\sigma = \frac{F}{A} = \frac{4\ \#}{16\ in^2} = 0.25\ psi$

Example Problem

What is the stress in the member at points A, B, and C? Notice that we won't be using the quotes around the points A, B, and C from now on. There is no need, since we all understand what we're talking about.

Solution Cut the member at points A, B, and C and draw three free-body diagrams.



Cut the member at point A and draw a free-body diagram.

Force at point A: $F = 300\# \uparrow$

Area of member at point A: $A = 3'' \text{ x } 3'' = 9 in^2$

Stress at Point A: $\sigma = \frac{F}{A} = \frac{300\ \#}{9\ in^2} = 33.3\ psi$

Cut the member at point B and draw a free body diagram.

Force at point B: $F = 300\# \uparrow$

Area of member at point B: $A = 4'' \text{ x } 4'' = 16 in^2$

Stress at Point B: $\sigma = \frac{F}{A} = \frac{300\ \#}{16\ in^2} = 18.8\ psi$

Cut the member at point C and draw a free body diagram.

Force at point C: $F = 300\# \downarrow$

Area of member at point C: $A = 5'' \text{ x } 5'' = 25 in^2$

Stress at Point C: $\sigma = \frac{F}{A} = \frac{300\ \#}{25\ in^2} = 12\ psi$

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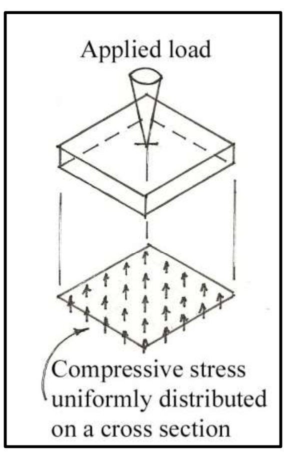
Notice in this case, we only drew the upper portion of the column (free-body diagram) for points A and B. And the bottom free-body diagram for point C.

Also, in this example problem, the cross sectional area of the column varies along the length of the column. And the load remains constant along the length of the column. Therefore the stress in the member varies along the length of the column.

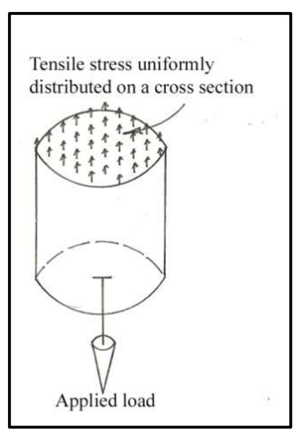
Stress Distribution

In all of the above example problems (which were axial loads along the long axis of a member), the compressive force is assumed to act uniformly over the entire cross section of the member creating a uniform stress. If the force varies across the cross sectional area of the member, it will create a non-uniform stress. (We'll deal with non-uniform stresses in the next course. Right now, let's just understand the concepts of uniform axial stress.) The exact same assumption is also true for tension members - the tensile force is uniformly distributed across the entire face of the cross section. The diagrams below are useful in visualizing the stress acting uniformly over the cross section.

Note: The assumption of uniform stress across the cross section is one reason that we only use three significant figures for the answer to problems. In real life, the loads (and, therefore, stresses) are not constant over the entire cross section. They are not uniformly distributed. They vary from some maximum to some minimum. However, to solve a problem with non-uniform varying loads is a complex mathematical calculation. To simplify the calculation, assume the loads are uniform. And, therefore, use only three significant figures in the answer.



Compressive Force



Tensile Force

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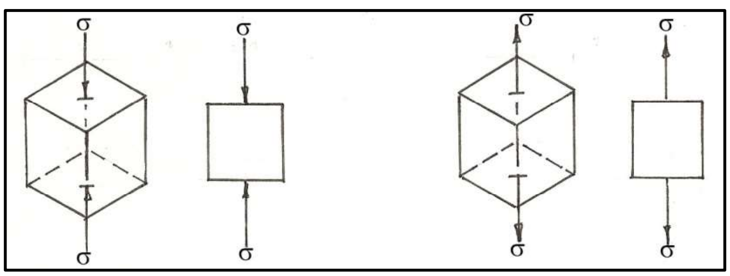
In other cases it is more convenient to visualize the stress acting on only a very small unit of the member - an infinitesimal element. Consider a very small element inside the square compression member shown above. If an element is taken from a body in equilibrium, the element is also in equilibrium. Therefore, there must be a compressive stress on the top of the element and a compressive stress on the bottom of the element. The three-dimensional cube (element) is the complete unit.

Instead of a three-dimensional figure, the unit is often shown as a two-dimensional figure with a force on top and a force on the bottom. If the element is taken to be a unit area (an infinitesimally small area), the force produces a unit stress on the element as shown below. Similarly, for the round member in tension, an element of unit area taken from the member will have a tensile force on the top and bottom faces. This force will then produce a unit stress on the element as shown below.



Element in compression   Element in Tension

SHEAR STRESS

There is another kind of stress besides axial stress. It is called **shear stress.** Shear stress is neither tension nor compression. Nor is it an axial load. It is more like a slipping or sliding of a member across itself.

Single shear

Single shear is the name given to a shear stress where only one plane through the member is subjected to the shearing force. The simplest example of single shear stress is the shearing of a bolt. The drawing on the next page shows two plates bolted together with a single bolt through both plates. The plates are being pulled by a force F. The plates each exert a force on the bolt that is perpendicular to the long axis of the bolt (and, therefore, is not an axial load). The action of the forces on the bolt tend to cause the bolt to slip or slide across the cross section of the bolt. This action on the bolt is single shear - also called direct shear stress.

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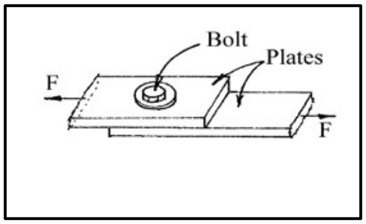
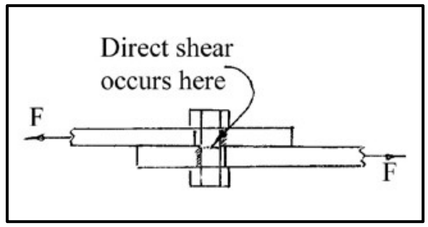
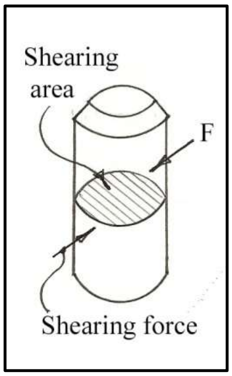






If we isolate the bolt, and show only the forces acting on the bolt, we can easily see and compute the direct shear stress on the bolt. In the adjacent drawing the applied load, F, is transferred to the bolt through the plates. Only the forces F are acting on the bolt. The bolt material will resist slipping or sliding along the shear plane. This resistance is the shear stress. The forces on the bolt are assumed to be resisted uniformly across the cross section of the bolt.

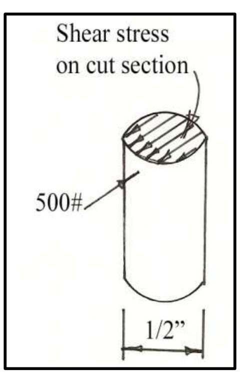
Shear stress is denoted by the lowercase Greek letter tau (τ). Direct shear stress is calculated by dividing the total force (F) on the cross section by the cross sectional area (A) resisting the force.

$$direct\ shear\ stress = \tau = \frac{applied\ force}{shear\ area} = \frac{F}{A}$$

Example Problem

Using the two bolted plates as an example, draw a free-body diagram of the bolt. Pass a plane through the bolt at the location of the direct shear stress and draw only the lower portion of the bolt. Show the force from the plate acting on the bolt and the resultant (resisting) direct shear stress on the cut section.

Let's say the bolt is 1/2 inch in diameter and the force pulling on the plates is 500 pounds. What is the shear stress in the bolt? (Note: 1,000 pounds = 1 kip. Sometimes it makes more sense to show an answer in kips per square inch rather than pounds per square inch.)



First, calculate the cross sectional area of the bolt over which the shearing force acts.

$$shear\ area = A = \frac{\pi D^2}{4} = \frac{\pi (0.50\ in)^2}{4} = 0.196349\ in^2$$

Then calculate the shear stress in the bolt at the cut section.

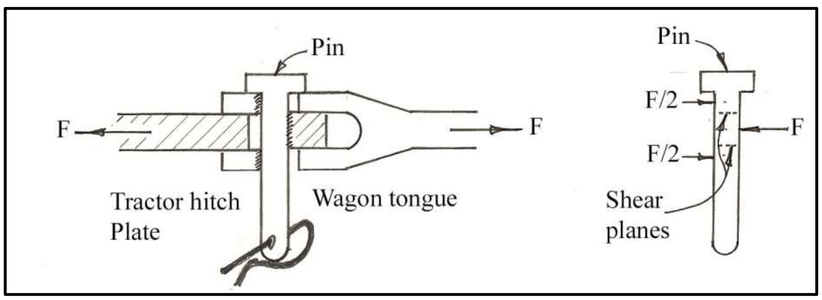
$$shear\ stress = \tau = \frac{F}{A} = \frac{500^{\#}}{0.196349\ in^2} = 2546\ psi \ \Rightarrow\ 2550\ psi = 2.55\ ksi$$

Notice in the above calculation that the shear area calculation was taken to six decimal places (six significant figures) while the answer was only taken to three significant figures. The six decimal result is a precision in arithmetic. The three significant figure answer represents the validity of the answer. In most real world problems, only the first few digits of an answer are valid, or "significant". The answer cannot be more accurate than the least accurate number used in the statement of the problem. Answers in Strength of Materials are commonly only taken to three significant figures.

Double Shear

Double shear occurs when a bolt or pin resists the externally applied load through two cross-sections of the member. Since the bolt in the above example is being sheared along only one plane, it is called single shear.

It is common to have a bolt or pin in double shear - for example hitching a wagon tongue to a tractor puts the pin in double shear. In the drawing below, a pin is dropped through the holes in the wagon tongue and the hole in the tractor hitch. As the tractor pulls the wagon, the pin is subjected to shearing forces along two cross sectional planes.



The tractor hitch is often a single bar with a slightly oversized hole (so the pin will drop through easily). The wagon hitch is often an upper and lower bar with oversized holes. The upper and lower bar allows the wagon tongue to bounce around (up and down) and not pull the pin out. Sometimes the pin has a "key" in the end to keep it in place. Other times a pin is simply dropped through the holes and gravity holds it in place.

Example Problem

If the tractor is pulling with a force of 500 pounds, and the connecting pin has a 3/8th inch diameter, what would be the shear stress in the pin in the drawing above?

The pin is resisting the tractor pulling force with two cross sections. The shear area is the sum of the two cross sectional areas as shown in the bolt free-body diagram above.

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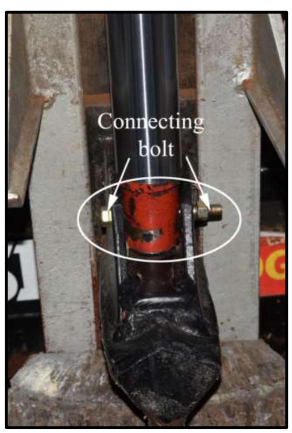
$$A = 2\left(\frac{\pi D^2}{4}\right) = 2\left[\frac{\pi\left(3/_8\ in\right)^2}{4}\right] = 0.22089\ in^2$$

And, the shear stress is calculated as:

$$\tau = \frac{F}{A} = \frac{500\ \#}{0.22089\ in^2} = 2{,}264\ psi \quad \Rightarrow \quad 2260\ psi = 2.26\ ksi$$

In the two examples above, we are assuming that the bolt and pin have sufficient shear strength to resist the direct shear stresses induced by the loads. We assume the bolt and pin will not fail. We'll get to the allowable shear stress and bolt capacities in a bit. But first, here is a real example of a bolt failing in double-shear.

I was splitting wood with a hydraulic log splitter and got the splitting wedge stuck in a knotted piece of wood. The wedge would not penetrate the wood any further, nor would the wood split to release the wedge. As is common, the bolt used to connect the hydraulic ram to the splitting wedge is not strong enough to resist the full force of the hydraulic ram. This is done to prevent damage to the hydraulic system or to the wedge. The inexpensive and easy to install bolt is designed to fail in shear before causing damage to the expensive equipment. Instead of backing off the power, I decided to push ahead and continue loading the connection to see if the bolt really would fail first. Just as expected, the bolt sheared and failed. It was exciting! (I did it in the name of science. That's my story and I'm sticking to it ☺)



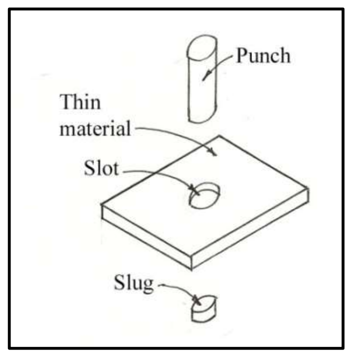
Notice in the bolt photo that the bolt is bent in addition to being broken. That means that the bolt did not fail in PURE shear. There were some bending stresses too. Notice also that the loads are shown as point loads when in fact they are distributed loads. And notice the crushing of some of the threads of the bolt. However, the assumptions made that it did fail in pure shear with point loads are still valid. These are reasons for using only three significant figures in the answer.

Cutting Shear

There are other times when we want the material to fail in direct shear. One example is the shearing action used by a sheet metal worker. When making sheet metal ducts, the worker uses a pair of shears similar to scissors to cut pieces of thin sheet metal. One blade of the shears slides over the other blade to cut the material. The shearing action progresses along the sheet metal cutting only a small part of the total cut at a time. The objective is to cut the material. You want the material to fail in shear.

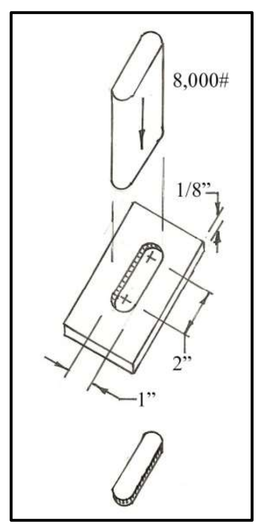
The cutting action of the sheet metal worker is exactly the same action of the single and double shearing action of the bolts above. The material - either bolt or sheet metal - is subjected to either a slipping or sliding action when an external load is applied.

Punching Shear



Another example where the end result is to fail a material in direct shear occurs in a punching operation. The goal of a punching operation is to create a hole in a sheet of material. The material fails in shear - hence the name **punching shear**. A punch is pushed through a thin sheet of material creating a hole called a slot. The piece of material removed from the sheet is called the slug. Sometimes it is the slug that is wanted - other times it is the slot that is the end product of the punching operation.

In punching operations the applied force of the punch is resisted by the area of the part actually being sheared. That area is the portion of the material that fails - the surface area of the cut portion of the material. The area of the edge of the slug, or the area of the edge of the slot. This area is calculated as the perimeter distance of the slug times the thickness of the slug. Which, of course, also equals the perimeter distance of the slot times the thickness of the material being punched. The shear stress is then calculated as:



$$\tau = \frac{Force}{Area} = \frac{F}{A}$$

Example Problem

In the adjacent drawing, what is the shear stress in the material if a force of 8,000 pounds is required to punch out the slug?

The shear area is the perimeter of the slot times its thickness.

$$A = perimeter\ x\ thickness = p\ x\ t$$

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Notice the perimeter of the slot (and the slug) consists of a 1" x 2" rectangle and a 1" diameter circle.

$$perimeter = p = 2(2\ inches) + \pi(1\ inch) = 7.14159\ in$$
$$thickness = t = \frac{1}{8}\ inch$$
$$A = p\ x\ t = (7.14159\ in)\left(\tfrac{1}{8}\ in\right) = 0.89270\ in^2$$

and

$$shear\ stress = \tau = \frac{F}{A} = \frac{8{,}000\ \#}{0.89270\ in^2} = 8{,}962\ psi \ \Rightarrow\ 8{,}960\ psi = 8.96\ ksi$$

Shear Stress Element

An infinitesimally small cubic element of the material from the shear plane of the single shear and double shear problems above would look like the cube shown below. For example an infinitesimally small cube (shown on the next page) taken from the shear plane of the single shear bolt would have a horizontal shear stress acting toward the left on the top surface. For equilibrium of the cube in the horizontal direction, there must be an equal and opposite shear stress on the bottom of the cube. This is the slipping or sliding action characteristic of shear. Note that the shear stresses are shown as an arrow with only a "half" of an arrow head.

The two stress vectors - one on the top and one on the bottom - on the cube cannot act alone. Together they form a couple that tends to rotate the cube. The cube subjected to just the two horizontal shears - one in each direction - is not in equilibrium. For equilibrium, there must be an opposite and equal couple. This couple is created by the equal - and opposite - shear stresses developed on the vertical sides of the cube. These four shear stresses - two couples - maintain equilibrium of the cube.

An infinitesimally small cubic element of the material from the shear plane of the cutting shear and the punching shear problems above would also look exactly like the cube shown on the next page. The shear stresses shown would, however, be developed slightly differently. Instead of beginning with two horizontal shear stresses and then adding the vertical couple, we would begin with the vertical shear stresses and then add the horizontal couple for equilibrium. The end result is the same.

The stress element is often drawn in two dimensions as shown. Notice that the stress vectors shown on adjacent sides of the cube meet at the corner of the cube. The two dimensional view is useful in visualizing the stresses acting at a point within a material subjected to shear.

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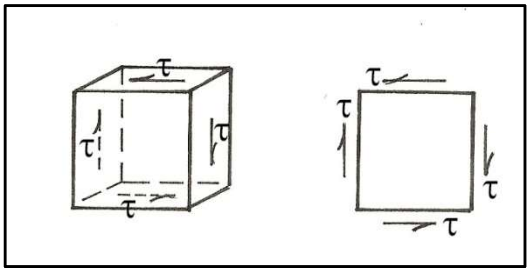


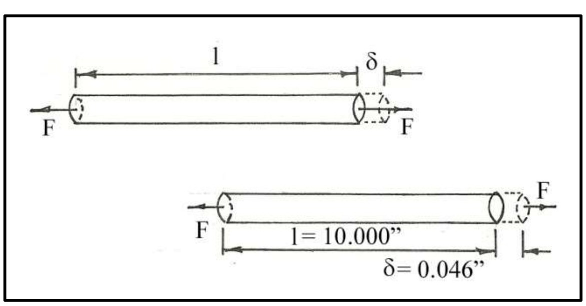
Also notice - and this is a biggie - that a horizontal shear stress (which is produced by horizontal forces) creates a vertical shear stress. Because there is a vertical shear stress, there must be vertical forces. And there are! Horizontal external forces on a bolt produce vertical forces (and, therefore vertical shears) in the bolt. It turns out that the vertical shear stress in the bolt is usually quite small compared to the horizontal shear stress because of the area over which the forces act. The horizontal force acts over an area determined by the diameter of the bolt while the vertical force acts over an area determined by the length of the bolt. And in most cases the bolt is much longer than it is in diameter. In the usual case of bolts, the vertical shear stress is ignored. However, look out for very large diameter, very short bolts. The vertical shear just might govern.

In the case of some beams, the created shear acting perpendicular to the load is critical. For example a horizontal beam with a vertical load produces a horizontal shear that can be considerable. We will deal with those stresses in the next course.

STRAIN REVEALED

Any structural member under tensile or compressive stresses will deform. Members under compressive stress will get shorter (and thicker). Members under tensile stresses will get longer (and thinner). The total deformation of a member under load can, of course, be measured. It can also be predicted. The "thinner" and "thicker" deformations are usually ignored.

The drawing below shows a bar subjected to an axial tensile load "F". The original length of the bar is denoted as a lowercase "L" (l). The total deformation of the bar, i.e., the increase in length of the bar, is denoted as a lowercase Greek letter delta (δ).



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Let's say that the original length (l) of the bar is 10.000 inches. And the total deformation (δ) is 0.046 inches. In other words, before the load F is applied, the total length of the bar is 10.000 inches. And after the load F is applied the total length of the bar is 10.046 inches.

Strain, also called unit deformation, is found by dividing the total deformation by the original length of the bar. The strain, or unit deformation, is denoted by a lowercase Greek letter epsilon (ε).

$$strain = \varepsilon = \frac{total\ deformation}{original\ length} = \frac{\delta}{l}$$

Example Problem

What is the strain for the rod shown above?

$$\varepsilon = \frac{\delta}{l} = \frac{0.046\ in}{10.000\ in} = 0.0046\ \ in/in$$

Since the inch per inch units could be cancelled during a calculation, the strain could be considered dimensionless. However, it is better to leave the units as in/in (or mm/mm) to maintain the definition of strain as deformation per unit length of the member.

MATERIAL PROPERTIES

In the design or analysis of a structural member, it is important to understand the properties of the material being considered. For concrete and steel, these properties are discussed in detail in the three SunCam courses titled **Fundamentals of Concrete; Fundamentals of Steel - Part A;** and **Fundamentals of Steel - Part B.**

Real Materials Have Limited Strengths

All real materials have an upper limit on their strength. When a material reaches its upper limit, its **ultimate strength**, it will fail. It will no longer be able to carry its intended load.

Failure can come in three different ways:

1. it may fracture (tear in tension - crush in compression);
2. it may stretch, or deform, excessively and no longer able to perform properly; or
3. the member may buckle becoming unstable and no longer able to carry its load.

Whatever the mode of failure may be, as the forces in the material continually increase, the member eventually reaches its upper limit, its maximum load carrying capacity, and fails.

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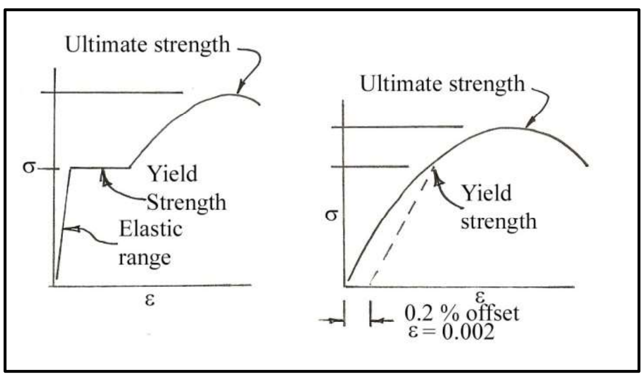
Of course, different materials have different strengths. For example steel can have an ultimate strength of around 250,000 psi while the ultimate strength of concrete is in the range of a few thousand psi. And some materials have different strengths when loaded in different directions. For example wood is stronger when loaded parallel to the grain than when loaded perpendicular to the grain. And concrete is stronger in compression than it is in tension.

Stress - Strain Diagrams

The stress-strain diagram of a particular material shows several components of the materials strength properties. One of these properties is the **ultimate strength** of the particular material. The ultimate strength of a material is the high point on its stress-strain diagram - its ultimate stress at which failure occurs.. The stress-strain diagrams of steel and concrete are shown below (in their general shape) with the ultimate strength noted.



Mild Steel Concrete

The stress-strain diagram also shows the **yield point** of a material. The yield point is the stress at which the material begins to deform without an apparent increase in load. For many steels, the yield point is quite prominent as shown in the diagram above. In materials such as concrete, where there is no obvious yield point, a **yield strength** is determined. By plotting a line approximately parallel to the straight line portion of the curve beginning at maximum acceptable strain, usually 0.2% ($\varepsilon = 0.002\ in/in$), a yield strength is determined. In practice, yield point and yield strength are used for the same purposes. And the terms are often used interchangeably.

The ultimate strength and yield point (yield strength) for materials are almost always listed in reference books on the mechanical properties of materials.

RELATIONSHIP OF STRESS AND STRAIN

A material behaves elastically when its stress levels are lower than the yield point. That is, at stresses below the yield point the material will return to its original size and shape after the load

is removed. The portion of the stress-strain diagram showing stress below the yield point is essentially a straight line. Way back in 1678, a scientist named Robert Hooke discovered that materials in the elastic range have a definite - and constant -relationship between the stress and the strain. Stress is proportional to strain in a particular material. His relationship became known as **Hooke's Law**. Hooke's Law states that there is a straight line relationship between the stress and the strain :
(σ) (ε)

$$\sigma = E\epsilon \qquad E = \frac{\sigma}{\varepsilon}$$

All of our discussions of Strength of Materials will be within the **elastic range** of the material. That means that if a material deforms a bit under a load, it will return to its original size and shape after the load is removed. We will not deal with stresses and strains that cause a material to exceed its elastic limit (its yield point). When materials exceed their elastic limit or yield point, they are considered either permanently deformed or deformed beyond their useful limit and the concepts here do not apply.

The **modulus of elasticity, E,** is the slope of the line of the stress-strain diagram in the elastic range.

$$E = \frac{change\ in\ stress}{change\ in\ strain} = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\sigma}{\varepsilon}$$

The modulus of elasticity is a fairly large number. For example, if a material requires a stress of $30{,}000\ psi$ to achieve a strain of $0.001\ in/in$, the modulus of elasticity of that material would be 30 million pounds per square inch as shown below.

$$E = \frac{\sigma}{\varepsilon} = \frac{30{,}000\ psi}{0.001\ \ in/in} = 30{,}000{,}000\ psi = 30x10^6\ psi = 30x10^3\ ksi$$

The modulus of elasticity is different for different materials.

<u>Steel</u>

Steel is a homogenous material which is manufactured in an environment where strict quality control measures are enforced. It is a nearly uniform material. As such, all commonly used structural steels have the same modulus of elasticity. The American Institute of Steel Construction specifies the modulus of elasticity for steel as 29 x 10^6 psi.

<u>Concrete</u>

The modulus of elasticity of concrete varies as the ultimate strength of the concrete mix and as the specific weight of concrete. According to the American Concrete Institute , the modulus of elasticity (E_c) of concrete can be estimated as $E_c = 33\gamma^{2/3}\sqrt{f_c'}$, where γ = unit weight of

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concrete, and f_c' is the ultimate strength of the concrete mix. For common construction grade concrete, the nominal value of the unit weight of concrete is 150 #/ft^3. Normal values for the ultimate compressive strength of concrete varies from around 2,000 psi to around 7,000 psi. The tensile strength of concrete is only a few hundred pounds per square inch - therefore it is usually assumed to be zero. (For concrete structures where the concrete members have portions of their cross sections in tension, reinforcing steel is placed in the member to resist the tensile forces.)

Below is a table showing the results of the ACI formula for E_c using γ = 150 #/ft^3 and various values of the ultimate strength of concrete, f_c'.

Modulus of elasticity for concrete	
Strength of concrete, f_c', psi	Modulus of elasticity, E_c, psi
2000	2.7 x 10^6
3000	3.3 x 10^6
4000	3.8 x 10^6
5000	4.3 x 10^6
6000	4.7 x 10^6
7000	5.1 x 10^6

Wood

The modulus of elasticity of wood (just as with strengths of concrete) also varies from species to species. Each species of wood and each grade within that species has a different modulus of elasticity. The table below and continued on the next page shows the modulus of elasticity for a few species of wood, and a few grades within each species.

Modulus of elasticity for wood	
Type and grade	Modulus of elasticity, psi
Douglas fir - 2 to 4 in thick, 6 in and wider	
No. 1	1.8 x 10^6
No. 2	1.7 x 10^6
No. 3	1.5 x 10^6
Hemlock - 2 to 4 in thick, 6 in and wider	
No. 1	1.5 x 10^6
No. 2	1.4 x 10^6
No. 3	1.2 x 10^6

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Southern pine - $2\frac{1}{4}$ to 4 in thick, 6 in and wider	
No. 1	1.6×10^6
No. 2	1.3×10^6
No. 3	1.3×10^6

When a material obeys Hooke's Law - i.e., stresses stay below the yield point (elastic limit) - the following is true:

$$E = \frac{\sigma}{\varepsilon}$$

If the member is subjected to an axial load (F), we can substitute for stress $(\sigma = \frac{F}{A})$ and strain $\left(\varepsilon = \frac{\delta}{l}\right)$ as follows:

$$E = \frac{F/A}{\delta/l}$$

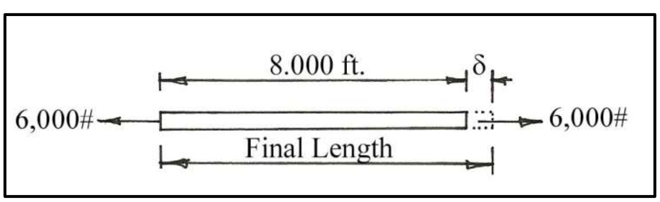
Solving for δ:

$$\delta = \frac{Fl}{AE} \Rightarrow \frac{(\#)(in)}{(in^2)(\#/in^2)} \Rightarrow in$$

This formula gives the relationship for deformation, load, length of member, cross sectional area of the member, and the modulus of elasticity for a material loaded axially. The member must have a constant cross sectional area, and the stresses must be under the elastic limit.

Example Problem

A 1/4 inch diameter steel cable is 8.000 feet long. If it is subjected to a tensile force of 6,000 pounds, what will be the final length of the cable? The modulus of elasticity of steel is 29 x 10^6 psi.



First determine the total elongation of the cable after the load is applied. Then add the elongation to the original length to determine the final length. All units in the calculation must

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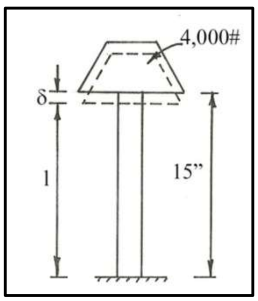
be consistent, therefore the 8 feet must be converted to inches. Also remember that the area of a circle is πr^2 or $\pi \frac{D^2}{4}$ where $r = \frac{D}{2}$

$$\delta = \frac{Fl}{AE} = \frac{(6{,}000\ \#)(8\ ft\ x\ 12\ {}^{in}/_{ft})}{\left[\pi\left(\frac{0.25\ in}{2}\right)^2\right]\ (29x10^6\ {}^{\#}/_{in^2})} = \frac{576{,}000\ \# - in}{1{,}423{,}532\ \#} = 0.405\ in$$

The final length of the loaded cable is then 8 ft plus 0.405 inches equals 96.405 inches.

Example Problem

A 2" x 2" block of wood is 15 inches long. Its modulus of elasticity is 1.4 x 10^6 psi. A load of 4,000 pounds is set on top of the block. What is the final length of the block of wood? Assume the block only shortens - it does not buckle.



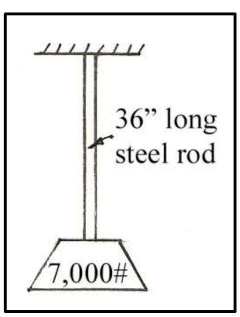
Calculate the total deflection of the block and then subtract it from the original length.

$$\delta = \frac{Fl}{AE} = \frac{(4{,}000\ \#)(15\ in)}{(2\ in\ x\ 2\ in)\ (1.4x10^6\ {}^{\#}/_{in^2})} = \frac{60{,}000\ \# - in}{5{,}600{,}000\ \#} = 0.011\ in$$

The final length of the block of wood is 15 inches minus 0.011 inches equals 14.989 inches.

Example Problem

A 36 inch long round steel rod is to support a hanging load of 7,000 pounds (axial force - tension). The modulus of elasticity of the rod is 29 x 10^6 psi. What minimum diameter rod is required so that the stress in the rod is ≤ 20,000 psi AND the total elongation of the rod is ≤ 17 x 10^{-3} inches (0.017 inches)?



The solution involves checking two requirements - the first requirement is the minimum diameter required to keep the stress less than or equal to 20,000 psi, and the second requirement is the minimum diameter required to keep the maximum elongation less than or equal to 0.017 inches. The larger diameter governs the selection. We will first determine the minimum cross sectional areas that meets each requirement; select the larger of the two; and then calculate the diameter for that area.

For stress, the minimum cross sectional area is:

$$A = \frac{F}{\sigma} = \frac{7{,}000\ \#}{20{,}000\ \ \#/in^2} = 0.3500\ in^2$$

The cross sectional area of the rod must be at least 0.3500 in^2. If the area is less than that, the stress in the rod would be greater than 20,000 psi. And would not meet the stress requirement.

For total elongation, the minimum cross sectional area is:

$$A = \frac{Fl}{\delta E} = \frac{(7{,}000\ \#)(36\ in)}{(0.017\ in)(29\ x\ 10^6\ psi)} = 0.5112\ in^2$$

The cross sectional area of the rod must be at least 0.5112 in^2. If the area is less than that, the elongation of the rod would be greater than 0.017 inches. And would not meet the elongation requirement.

The minimum diameter rod that meets both the stress and the total elongation requirements is a round rod with a cross sectional area of at least 0.5112 in^2. The diameter of a rod with a cross sectional area of 0.5112 in^2, or greater is:

$$A = \frac{\pi\, D^2}{4} \quad and\ \ D = \sqrt{\frac{4\ A}{\pi}}$$

$$Minimum\ required\ diameter\ rod = D = \sqrt{\frac{4\ A}{\pi}} = \sqrt{\frac{(4)(0.5112\ in^2)}{\pi}} = 0.807\ in$$

To the nearest sixteenth of an inch, a rod with a 13/16 inch (0.8125") diameter would suffice. Or, if that size was not conveniently available, a 7/8 inch (0.875") diameter rod would work. It would support the 7,000 pound load with a maximum stress of less than 20 ksi and a deformation of less than 0.017 inches.

DESIGN STRESS

We have been working with elongation due to direct stress on a member - either tension or compression. Several assumptions are inherent in the formula for direct stress ($\sigma = F/A$).

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Some Assumptions
The loaded member:

1. must be straight.
2. must have a uniform cross section over the entire length of the member.
3. must be made from a homogeneous material.
4. must be loaded along the centroidal axis of the member (along the center of gravity of the cross section).
5. in compression must be "short" so there is no tendency to buckle (we'll determine what "short" is later).

If all of these conditions are met, a member could be loaded up to its yield strength without fear of failure. And, in some cases, up to its ultimate strength without fear of failure. However, in real life, the above five conditions are seldom, if ever, met perfectly. An ever so slight bend in a member, or a load that is slightly offset from the exact center of gravity of the cross section, introduces bending stresses in the member (which are different from direct stresses - and, again, we'll get to bending stresses in another course). Also, no cross section is perfectly uniform. And no material is perfectly homogeneous. For example, the aggregate in concrete is not exactly uniform is size, shape, or strength, and, the water-cement paste has different properties than the aggregate. And there are imperfections in wood such as knots, checks, splits, cracks, etc. All of this, again, suggests a reasonableness to using only three significant figures in an answer to a strength of materials problem.

Failure
Failure occurs when a load carrying member breaks or deforms excessively, making it unacceptable for its intended purpose. Therefore, it is essential that the level of stress in the member never exceed the yield stress or, in some cases, the ultimate stress of the material. The yield strength and ultimate strength of materials is often given in table form in reference manuals. For example:

Structural Steel		
Grade of steel	Ultimate Strength σ_u (ksi)	Yield Strength σ_y (ksi)
A36 - shapes, plates, and bars	58	36
A242 - shapes, plates, and bars ≤ 3/4 inch thick	70	50
A514 - quenched & tempered alloyed steel: plate ≤ $2\frac{1}{2}$ inches thick	110	100

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Design Stress
To make sure a material (loaded member) doesn't fail in use, a maximum allowed **design stress** has been determined for each different material and for different loading conditions. The maximum allowed stress for design and analysis is determined by applying a factor of safety to either the yield strength of the material or the ultimate strength of the material.

$$maximum\ allowed\ design\ stress = \frac{yield\ strength}{Factor\ of\ Safety}\quad OR\quad \frac{ultimate\ strength}{Factor\ of\ Safety}$$

A different factor of safety (FS) is applied to each different material to account for its consistency. For example, steel is more consistent than say, wood, because of the high standards of quality control during steel's manufacture. We have a lot less control over the "manufacture" of wood material - it grows in nature with all the irregularities that that brings, many of which we never see (irregular grain pattern, tiny knots, etc.). Therefore, steel has a lower factor of safety, giving it a higher percentage of its yield strength or ultimate strength as an allowed design stress. Whereas wood has a higher factor of safety, giving it a lower percentage for its allowed design stress.

Working Stress and Allowable Stress
The maximum allowed design stress is also called the **working stress** or **allowable stress** and is given by the formulas:

Based on yield strength -

$$allowable\ stress = working\ stress = \frac{yield\ strength}{Factor\ of\ Safety} = \frac{\sigma_y}{FS} = \frac{s_y}{FS}$$

Based on ultimate strength -

$$allowable\ stress = working\ stress = \frac{ultimate\ strength}{Factor\ of\ Safety} = \frac{\sigma_u}{FS} = \frac{s_u}{FS}$$

Allowable stress and working stress are often used interchangeably. And, depending on which area of structural engineering you're working in (e.g., buildings, construction equipment, automotive, aerospace, etc.), and which material you're working with, different symbols are used for the yield strength and ultimate strength of a material. Two symbols - σ and s - are shown in the formulas above to indicate stress, and are used mostly for metals, composite materials, and plastics. When working with wood, F is used to indicate stress.. And, for those working in the field of concrete design and analysis, the ultimate strength - which is called the ultimate compressive stress (28 day strength) - is represented by the symbol f_c' .

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Whatever the symbol used or the name given, the concept of allowable and working stress is the same - which is the upper limit divided by a factor of safety.

Factor of Safety

The **factor of safety (FS)** to be used for determining the allowable stress (working stress - design stress) in a particular material and under certain loading conditions has been determined over the years through testing, research, and trial and error by lots of smart people with lots of experience in the area. Special interest groups also determine the FS for their material - e.g., American Institute for Steel Construction for steel; American Concrete Institute for concrete, etc.

Some factors that influence the factor of safety are:

- physical size of the material - thicker or larger pieces usually have a higher FS, and therefore lower design stresses, than thinner pieces;
- industry in which you are working - aerospace has smaller FS than construction equipment;
- type of loading - The FS is higher for impact loading than it is for static loading;
- the consequence of failure - the frame for an office building can have a higher FS (lower design stress) than a temporary material storage building on a construction project;
- cost - sometimes compromises are made by designers to be competitive in the marketplace.

There may be other factors that influence the final selection of a factor of safety in a calculation.

Often, the designers of the most common structural elements will not themselves choose the factor of safety to be applied to the yield or ultimate stress of a material. Building codes and standards are authoritative resources for determining which FS to use when making a structural engineering calculation. If the project or member being designed falls under the jurisdiction of a particular building code or standard, then the FS presented in that building code or standard must be the minimum used.

A few of the codes and standards are:

- American Institute of Steel Constructions (AISC) - buildings, bridges, and similar structures using steel
- American Institute of Concrete (AIC) - concrete structures.
- Concrete Reinforcing Steel Institute (CRSI) - steel reinforced concrete structures.
- American Institute of Timber Construction - timber structures.
- American Society of Mechanical Engineers (ASME) - boilers, pressure vessels, etc.
- Department of Defense - military standards, aerospace vehicles, etc.

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- International Building Code (IBC) - for buildings involving public safety.
- American National Standards Institute (ANSI) - products from many different areas.

Often in the publications listed above, there is a discussion of factors of safety and how they are applied to various materials and loading conditions. You may or may not use this FS information. As a practical matter though, most of the publications simply list the allowable stress for a particular material and loading condition. The application of the factor of safety and the related calculation is done and the result is listed in table format.

Following are a couple of examples for axial loads. The first table shows that the ultimate strength and the yield strength of a particular steel alloy changes as the size of the specimen changes. As the size of the specimen gets bigger, the FS gets bigger and, therefore, the ultimate strength and the yield strength decrease. The second table shows the different factors of safety for different loading conditions on a single material.

Effect of Size on a Single Steel Alloy		
Size of specimen (inches)	Ultimate strength (ksi)	Yield strength (ksi)
0.50	158	149
1.00	140	135
2.00	128	103
4.00	117	87

Working Stress for Load Types	
Type of loading	Working stress
Static	$\sigma_y/2$
Repeated	$\sigma_u/8$
Impact	$\sigma_u/12$

Working stresses are often given in table form in reference books. In these cases, the factor of safety has already been applied to the ultimate strength or the yield strength, and different loading conditions have also already been taken into account. For example, some of the properties of wood are shown below and on the next page.

Properties of Wood Allowable Stress					
Type & grade	Bending (psi)	Tension parallel to grain (psi)	Compression parallel to grain (psi)	Compression perpendicular to grain (psi)	Modulus of elasticity (psi)
Douglas Fir - 2 to 4 in. thick, 6 inches and wider					
No. 1	1,750	1,050	1,250	385	1,800,000
No. 2	1,450	850	1,000	385	1,700,000

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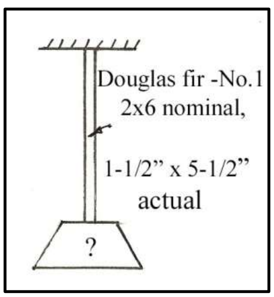
Hemlock - 2 to 4 in. thick, 6 inches and wider No. 1 No. 2	 1,400 1,150	 825 675	 1,000 800	 245 245	 1,500,000 1,400,000
Southern Pine - 2½ to 4 in. thick, 6 inches and wider No. 1 No. 2	 1,400 1,000	 825 575	 850 550	 270 230	 1,600,000 1,300,000

Example Problem

A nominal 2 x 6 piece of wood has actual dimensions of 1½" by 5½". Its cross sectional area is therefore $8.25in^2$. If the wood is Douglas Fir - No. 1, what is the maximum safe axial load that could be hung from the end of the 2 x 6?



Solution

The axial load on the 2 x 6 is tension parallel to the grain. Using the Properties of Wood table above, the allowable stress for Douglas Fir - No. 1 in tension parallel to the grain is 1,050 psi.

The formula for maximum load is derived from the stress formula as follows:

$$\sigma = \frac{F}{A} \;\Rightarrow\; F = A\sigma$$

Therefore:

$$maximum\ load = F = (area\ of\ cross\ section)(working\ stress)$$

$$F_{MAX} = (8.25\ in^2)\left(1{,}050\ {}^{lb}/_{in^2}\right) = 8{,}662.5\ lb = 8{,}660\ lb = 8.66\ kips$$

Example Problem

If the 2 x 6 in the above example was originally 10.000 feet long, how long would it be after the load was applied? From the table above, the modulus of elasticity of the wood is 1,800,000 psi (1,800 ksi).

Solution

The total deformation of a material is given by the formula:

$$\delta = \frac{Fl}{AE}$$

The total elongation of the 2 x 6 would then be:

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$$\delta = \frac{(8.66\ kips)[(10.000\ ft)(12\ in/ft)]}{(8.25\ in^2)(1{,}800\ \ kips/in^2)} = 0.06998\ in = 0.070\ in$$

And:

$$Total\ length = l + \ \delta = 120.000\ in + 0.070\ in = \ 120.070\ in$$

EFFECT OF TEMPERATURE

A change in temperature has an effect on materials. For nearly all common structural materials, if the temperature is raised, the material will expand. If the temperature is lowered, the material will contract. Different materials react to changes in temperature differently. All materials do not expand and contract at the same rates. The rate at which a specific material will expand or contract is given by its **coefficient of thermal expansion**.

The coefficient of thermal expansion, identified by the lower case Greek letter alpha (α), is defined as:

$$\alpha = \frac{change\ in\ length}{(original\ length)(change\ in\ temperature)} = \frac{\delta}{(l)(\Delta T)}$$

Stated slightly differently, alpha (α) is a measure of change in length of a material per unit of length for a 1-degree change in temperature. The units for α then would be:

$$\alpha = \frac{\delta}{(l)(\Delta T)} = \frac{in}{(in\ \cdot\ °F)} = \frac{1}{°F} = \ °F^{-1}$$

The units for the coefficient of thermal expansion in the metric system would be $℃^{-1}$.

The coefficients of thermal expansion have been determined for various materials and are listed in reference books just as other material properties are listed. The coefficients of thermal expansion for a few materials are listed in the following table:

Coefficient of Thermal Expansion, (α)		
Material	$°F^{-1}$	$°C^{-1}$
Structural steel	6.5×10^{-6}	11.7×10^{-6}
Concrete	6.0×10^{-6}	10.8×10^{-6}
Wood (pine)	3.0×10^{-6}	5.4×10^{-6}
Brass, C36000	11.4×10^{-6}	20.5×10^{-6}
Copper, C14500	9.9×10^{-6}	17.8×10^{-6}
Plate glass	5.0×10^{-6}	9.0×10^{-6}

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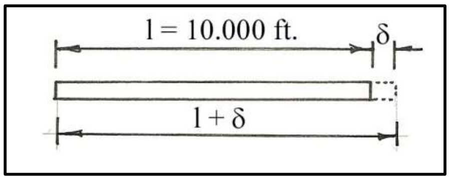
The formula for the change in length of a member caused by a change in temperature can be derived as follows:

$$\alpha = \frac{\delta}{(l)(\Delta T)} \quad \Rightarrow \quad \delta = \alpha \cdot l \cdot \Delta T$$

The change in temperature, ΔT, is given in either °F or °C depending on the units of the problem (English or metric) and the corresponding α must be used - either $°F^{-1}$, or $°C^{-1}$.

Example Problem

If a structural steel bar is 10.000 ft long at 0°F, how long would the bar be at 100°F?



Solution

First, solve for the total elongation of the bar due to the temperature change, and then add it to the original length. From the table above, the coefficient of thermal expansion, α, for steel is 6.5 x 10^{-6} $°F^{-1}$, and the change in temperature is 100 degrees Fahrenheit.

$$\delta = \alpha \cdot l \cdot \Delta T = (6.5\, x\, 10^{-6}\, °F^{-1})[(10\, ft)(12\ in/ft)](100°F) = 0.078\, in$$

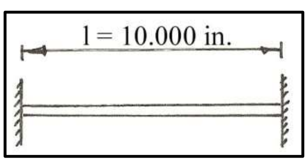
And the total length would be the original length plus the change in length due to temperature change:

$$original\ length + change\ in\ length = l + \delta = 120.000 + 0.078 = 120.078\, in$$

If a member is allowed to expand or contract freely during a temperature change, as in the example above, no stresses are induced in the member. The member has zero stress after the temperature change. If, however, a member is constrained in some way, and the temperature changes, then internal stresses (either tensile or compressive) are induced into the member. These stresses can be calculated.

Example Problem

A 1 inch by 1 inch brass bar is 10.000 inches long. It is constrained at both ends and not allowed to expand. If the temperature is raised by 100°F, what is the internal stress developed in the bar? The coefficient of thermal expansion, α, for brass is 11.4 x 10^{-6} $°F^{-1}$. The modulus of elasticity, E, for brass is 16 x 10^{6} psi.



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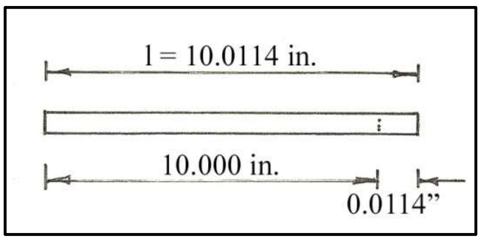
Solution

We will take two separate steps to solve this problem. The first step is to assume that the bar is free to expand and calculate the total length of the bar after expansion. At this point, there is no stress in the bar. The second step is to then apply a force to the "expanded" bar to "compress" it back to the original size. The force required to "shrink" the bar divided by the cross sectional area of the bar gives the internal compressive stress in the bar due to an increase in temperature with no expansion allowed.

First, calculate the total length of the expanded bar.

$$\delta = \alpha \cdot l \cdot \Delta T = (11.4\,x\,10^{-6}\,°\mathrm{F}^{-1})(10\,in)(100°\mathrm{F}) = 0.0114\,in$$

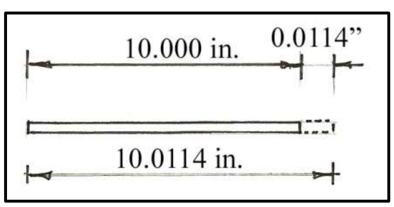
And the total length of the expanded bar is 10.0114 inches



Second, calculate the force required to shorten the bar back to its original length and calculate the stress in the bar. Actually, by rearranging the equation $= Fl/AE$, we can solve for stress directly as follows:

$$\delta = \frac{Fl}{AE} = \left(\frac{F}{A}\right)\left(\frac{l}{E}\right) = (\sigma)\left(\frac{l}{E}\right) \Rightarrow \sigma = \frac{\delta E}{l}$$

Solving this equation yields the stress in the fixed bar due to an increase in temperature of 100°F. The length (l) used in this equation is the "expanded" length due to the temperature change.



$$\sigma = \frac{\delta E}{l} = \frac{(0.0114\,in)(16\,x\,10^6\,psi)}{(10.0114\,in)} = 18{,}219.2\,psi \Rightarrow 18{,}200\,psi = 18.2\,ksi$$

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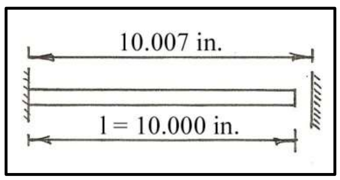
Notice in the solution to the problem that the cross sectional area of the bar was not used, nor was it needed, to determine the stress in the bar due to restricted movement during a change in temperature.

Sometimes the amount of expansion or contraction that is allowed to occur during a temperature change is less than the member needs to expand or contract without induced stresses. To solve for the stress in the material that is partially restrained after the temperature change, the same procedure is used as above except that when solving for the stress, the difference in length between the allowed and the "desired" length is used.

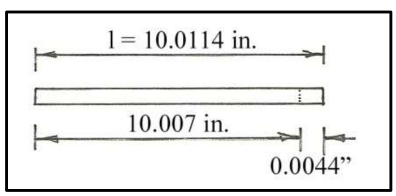
Example Problem

Use the data from the above problem. Except now change the total restricted length to 10.007 inches instead of 10.000 inches.



Step one is to allow the bar to expand unrestricted and calculate the length of the bar after the temperature change. The expanded unrestricted length (from the previous example above) is 10.0114 inches.

Step two in this problem is to allow the bar to only expand to 10.007 inches. Which means that from its unrestricted expansion of 10.0114 inches, it must be compressed back to 10.007 inches. It must be compressed a total of 0.0044 inches (10.0114 inches minus 10.0070 inches).



To compute the stress in the bar, the length to compress the bar is 0.0044 inches Calculate the stress as follows:

$$\sigma = \frac{\delta E}{l} = \frac{(0.0044\ in)(16\ x\ 10^6\ psi)}{(10.0114\ in)} = 7{,}032\ psi \Rightarrow 7{,}030\ psi = 7.03\ ksi$$

This makes sense. To raise the temperature of the bar 100°F and not allow it to expand causes an internal compressive stress of 18.2 ksi. If the bar is allowed to expand a bit, the internal compressive stress is lower (in this case the stress is lowered to 7.03 ksi).

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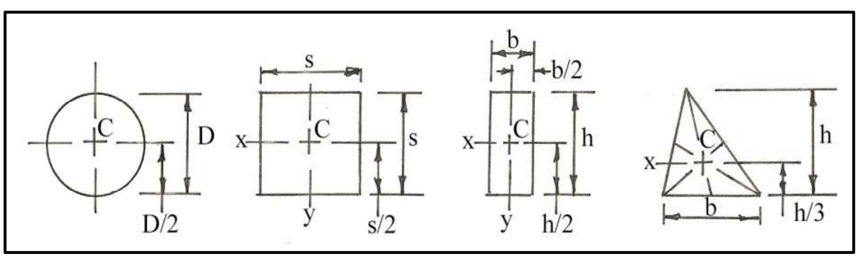
CROSS-SECTIONAL PROPERTIES

So far we have considered axial loads (tension and compression) where the load is acting along the long axis of the member, and direct shear loads (bolt shear, where the load acts perpendicular to the long axis of the member), and punching shear. In the axial load and direct shear calculations, the area of the cross section of the member is a key property of the cross section. It turns out that there are other very important properties of the cross section of a loaded member that are necessary for additional study of strength of materials. We will now learn how to determine one of these properties **-** the location of the centroid of an area.

CENTROID

The **centroid** of a cross section is the center of mass of that area. The cross section of a structural member (typically a beam or column) is a plane area and is two dimensional. The center of mass of a cross sectional area is a single point where the area could be balanced if it were supported only at that point.

For simple shapes, the centroid of the area is easy to see and to locate. Shown below are a circle, square, rectangle, and a triangle with their centroid shown. If each area were carefully cut out of a piece of cardboard, the piece could be balanced on the tip of your finger if your finger was placed at the centroid - the point on the drawings below defined as "C".



The location of the centroids above are almost intuitive, except maybe for the triangle. The centroid of the triangle is at the intersection of the three lines, each drawn from a corner of the triangle to the mid-point of the opposite side. A couple of rules present themselves for locating the centroid of an area.

- If a shape has only one axis of symmetry, the centroid will lie somewhere on that axis.
- If an area has two axis of symmetry, the centroid will lie at the intersection of the two axis.

The following shapes demonstrate the above rules. Each cross section below has two axis of symmetry, and, therefore, the centroid is located at the intersection of those two axis.

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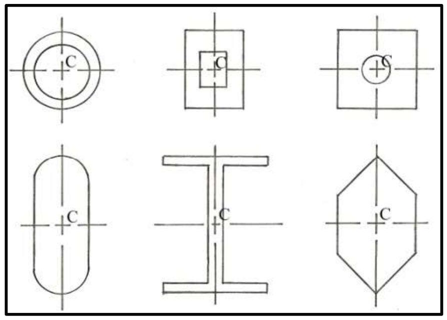

Often a structural member will have a cross section that is composed of two or more simple shapes. It may have one axis of symmetry, but not two. In that case, the centroid lies somewhere along the single axis of symmetry. And the location of the other axis must be determined.

NOTE: Let's mention here that in structural engineering the two axis of a cross section are labeled the x-axis and the y- axis. And they are perpendicular to each other.

By applying a simple set of arithmetic calculations we can determine where along the symmetrical axis the centroid is located. The procedure is called the method of composite areas.

Method of Composite Areas

For cross sections that do not have two axis of symmetry, the **method of composite areas** is used to locate the centroid of the area. The method of composite areas is sometimes called the **first moment of area**. Cross sections with only one axis of symmetry often contain two or more simple areas where their centroids are known (circle, square, etc.). Also, many cross sections will have at least one axis of symmetry in real life conditions.

The method of composite areas to find the centroid of an area consists of a few simple steps.

1. Break the complex area into simple areas that have known centroid locations (usually circles, squares, rectangles and triangles). These centroids are noted as "C" or (c.g.) on the following problem drawings.

2. Choose a reference axis about which to sum the moments of the simple, smaller areas.

3. Add the moments of the small areas about the reference axis. The distance from the reference axis to the centroid of each small area is denoted as "y". Therefore, the sum of the moments of the small areas (A_1, A_2, A_3, etc.) becomes $A_1y_1 + A_2y_2 + A_3y_3$ + etc. A hole in the cross section, or a void space, would be a negative area.

4. Find the distance from the reference axis to the centroidal axis ($\bar{y}$) by dividing the sum of the moments by the total area of the cross section. $\bar{y} = \frac{\text{A1y1 + A2y2 + A3y3 + etc.}}{\text{A1 + A2 + A3 + etc.}}$.

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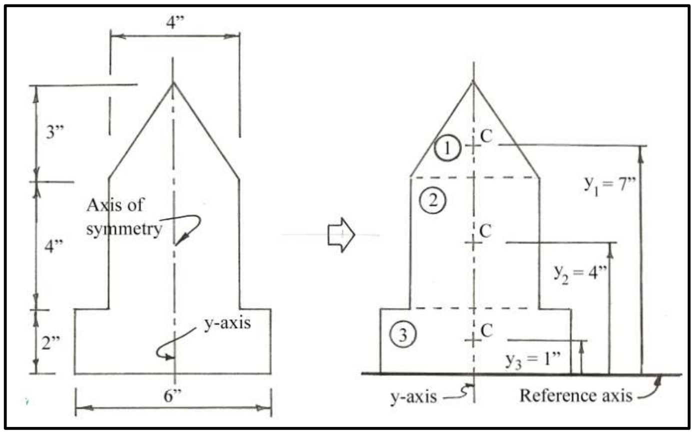
Example Problem

Find the location of the centroid of the cross section shown below.



Solution

Step 1 Break the cross section into three simple shapes and identify them as 1, 2, and 3. Locate the centroid (C), or center of gravity (c.g.), of each shape. Note: the y-axis is the axis of symmetry, therefore the centroid of each shape lies on that axis. The centroid of the entire shape lies on the y-axis as well.

Step 2 Choose a location for the reference axis. In this case, locate the reference axis at the base of the cross section. Compute the distance from the reference axis to the centroid of each small area as shown in the drawing.

Step 3 Compute the moment of each small area about the reference axis.

Area 1 ⇒ $A_1 y_1 = \left(\frac{1}{2} \cdot 4'' \cdot 3''\right)(7'') = 42\ in^3$

Area 2 ⇒ $A_2 y_2 = (4'' \cdot 4'')(4'') = 64\ in^3$

Area 3 ⇒ $A_3 y_3 = (2'' \cdot 6'')(1'') = 12\ in^3$

$\overline{118\ in^3}$

Step 4 Find $\bar{y}$ (which is the distance from the reference axis to the centroid of the entire section).

Calculate the total area of the cross section = 6 in^2 + 16 in^2 + 12 in^2 = 34 in^2.

$$\bar{y} = \frac{Sum\ of\ moments}{Total\ cross\ section\ area} = \frac{118 in^3}{34 in^2} = 3.47''$$

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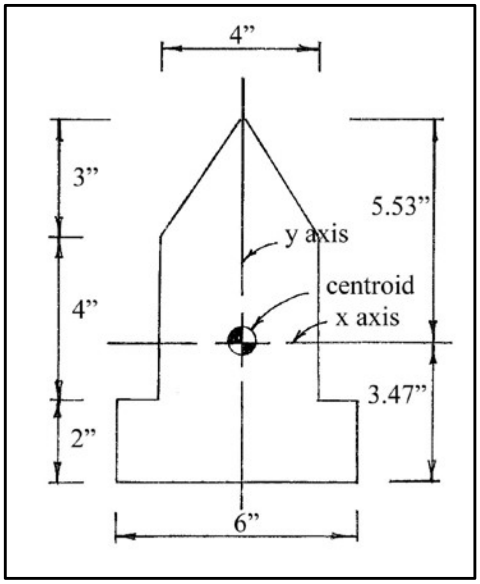
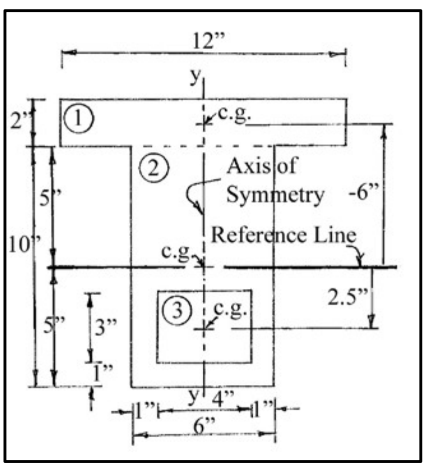


The centroid of the cross section is located along, or on, the y-axis of symmetry (the vertical axis), and the x-axis is located 3.47 inches from the reference axis as shown on the drawing to the left.

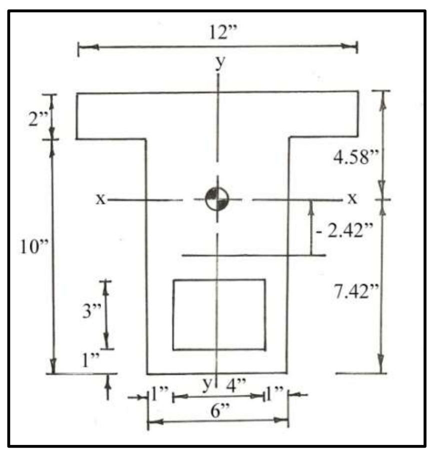
We'll work another example, this time putting our calculations in table format (easier to use), using the centroid of one of the smaller areas as the reference axis (simplifies one calculation), and inserting a hole in the cross section (to show a "negative area"). By choosing the reference axis as the axis of one of the simple shapes, we may also create a negative moment. And, we'll also show the three simple shapes and their centroids distance to the reference axis on the problem drawing.

Example Problem

Find the location of the centroid of the cross section shown below. Because the cross section is symmetric about the y-axis (the axis of symmetry), the centroid of the cross section lies somewhere along the y-axis.



Problem



Answer

Solution

Step 1 Break the cross section into three simple shapes 1, 2, and 3. Locate each shapes centroid (C) (or center of gravity - c.g.).

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Step 2 Locate the reference axis through the c.g. of small area 2. Show the distance from the

reference axis to the C of each small area. Choose the up direction from the reference axis as negative and down from the reference line as positive.

Step 3 and 4 Solve for $\bar{y}$ using a table format. In the table below, the dimensions are left off for simplicity. And the calculations are simple enough to do mentally. For example, the area of small area 1 is 2" by 12" equals 24 in^2. Area 3 is a void, therefore its area is negative. And y for area 1 is negative because it is a dimension <u>above</u> the reference axis, which we chose as the negative direction.

Area	A	y	Ay
1	24	-6	-144
2	60	0	0
3	-12	2.5	-30
	Sum = 72		Sum = -174

$$\bar{y} = \frac{-174}{72} = -2.42"$$

The negative sign simply means the x-axis is located above the reference axis. The centroid of the cross section is located along, or on, the y-axis of symmetry and the x-axis is located 2.42 inches above the reference axis.

The location of the x-axis (center of gravity or centroid of the cross section) also makes sense **intuitively**. By simply looking at the cross section before beginning the solution we can determine that the centroid will lie somewhere along the y-axis <u>AND</u> above the centroid of the middle small area - area 2. We do this by first visualizing in our mind only the 6" x 10" area. The centroid of that area lies at its midpoint - 5 " from the top of the rectangle and 5" from the bottom of the rectangle. Next, add the small triangular area - area 1 - to the top of the rectangle. This area moves the centroid up a bit. Then take some area that is below the centroid of the rectangle, away from the rectangle. Removing this area moves the centroid of the complete section slightly farther up. The final location of the x-axis will be somewhere along the y-axis and slightly above the centroid of the small rectangular area. In fact, it is 2.42 inches above the centroid of the rectangular area.

In real life, the above calculation method of locating the center of gravity or centroid of a cross section is usually limited to applications where simple shapes are combined to create a member. For example (see below), creating a Tee cross section by nailing a 2 x 6 to the underside of a 2 x 12 to create a 12 inch wide players bench for the sidelines of a high school football field. Or perhaps creating a wooden beam window header composed of a 2 by 12 with 2 by 6's nailed to

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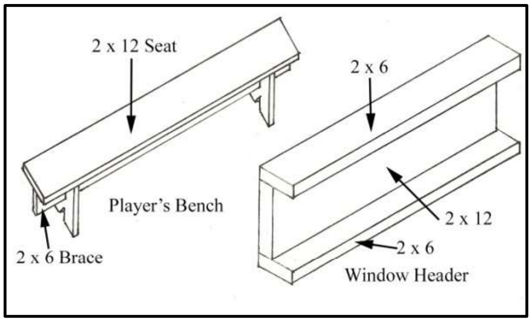
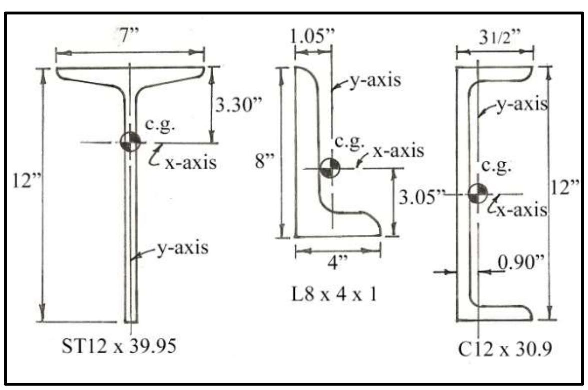
the top and bottom to form a "c" section. The 2 x 6's match the thickness of the exterior wall, and the space between the 2 x 6's can be filled with insulation.



Also, in real life, especially if you are working with steel, the shapes commonly used in structural systems are not simple areas. They may contain rounded corners, varying thicknesses of flanges, etc. The location of the centroids of these commonly used shapes - which are often quite tedious to compute - are often listed in reference manuals such as the Manual of Steel Construction published by the AISC.

Following are a couple of shapes found in the AISC Manual showing the listed dimension to the centroid of the cross section. Note that the dimensions of the cross sections change from time to time in the manual because of changes in the manufacturing process, or for other reasons. The different editions of the AISC Manual publish the current correct dimensions. Also, sometimes, a section will be discontinued. And will then be omitted from the following edition of the manual.

Dimensions shown on the drawings of the cross sections below are from the AISC Manual, 6th Edition, Second Revised Printing, November 1, 1965. The table below the drawings compares the 6th edition dimensions with those of the 8th edition of the AISC manual (1980).



6th Edition	y = 3.30"	x = 1.05"; y = 3.05"	x = 0.90"
8th Edition	y = 3.29"	x = 1.05"; y = 3.05"	x = 0.674"

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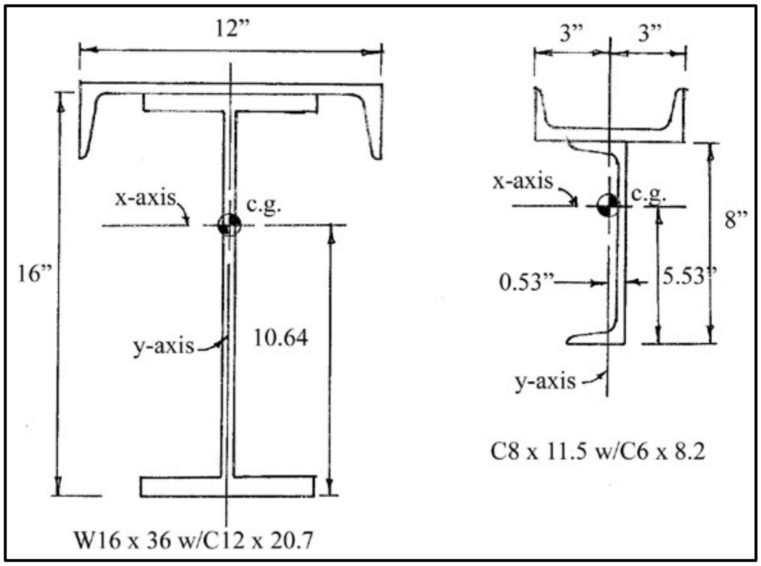

6th Edition	y = 10.64"	y = 5.53"
8th Edition	y = 10.67"	Not Shown in 8th Edition

This is the end of this course.

In the next course, **What Every Engineer Should Know About Structures - Part D - Bending Strength of Materials**, we will pick up where we left off here beginning with properties of an area known as the Moment of Inertia and the radius of gyration. Then we will get into torsional stresses and deformations, bending stresses including shear and bending moment diagrams, and deflections in beams.

The next course will be available soon.

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