

# **Illumination**

**Basics / Theory / Units & Conversions / Design Methods**

**by**

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## Illumination

### Nomenclature<sup>1</sup>

$A$	area	$\text{m}^2$
$b$	Wein's wavelength displacement constant	$2.8978 \times 10^{-3} \text{ m} \times \text{K}$
$b'$	Wein's spectral radiance displacement constant	$4.0956 \times 10^{-4} \text{ W/sr} \times \text{m}^2 \times \text{m} \times \text{K}^5$ or $\text{J/s} \times \text{sr} \times \text{m}^2 \times \text{m} \times \text{K}^5$
$c$	speed of light	$2.9979 \times 10^8 \text{ m/s}$
CCT	correlated color temperature	K
CIE	Commission Internationale de l'Eclairage (International Commission on Illumination)	—
CR	cavity ratio	—
CU	coefficient of utilization	—
$d$	distance	m
$D$	depth	m (ft)
DF	daylight factor	—
DOE	Department of Energy	—
$E$	energy	J
$E$	illuminance	lx (SI) fc (customary U.S.)
ERC	externally reflected component	—
$f$	frequency	Hz
FCR	floor cavity ratio	—
$h$	Planck's Constant	$6.6256 \times 10^{-34} \text{ J} \times \text{s}$
$h, H$	height	m (ft)
$I$	luminous intensity	cd
IRC	internally reflected component	—
$K$	luminous efficacy	lm/W
$\kappa$	Boltzmann's constant	$1.3807 \times 10^{-23} \text{ J/K}$
$L$	length or wavelength	m (ft)
$L$	luminance	lm
$L$	radiance	$\text{W/sr} \times \text{m}^2 \times \text{m}$
$L(\lambda, T)$ or $L_\lambda$	spectral radiance	$\text{W/sr} \times \text{m}^2 \times \text{m}$ or $\text{J/s} \times \text{sr} \times \text{m}^2 \times \text{m}$

<sup>1</sup> Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related, and may be found when reviewing the references listed for further information.

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$L(f, \lambda)$ or $L_f$	spectral radiance	$\text{W}/\text{sr} \times \text{m}^2 \times \text{Hz}$ or $\text{J}/\text{s} \times \text{sr} \times \text{m}^2 \times \text{Hz}$
$L_m$	maximum spectral radiance	$\text{W}/\text{sr} \times \text{m}^2 \times \text{m}$ or $\text{J}/\text{s} \times \text{sr} \times \text{m}^2 \times \text{m}$
LLF	light loss factor	—
$m$	mass	kg
$M$	luminous or radiant exitance	$\text{lm}/\text{m}^2$
$n$	index of refraction	—
NECA	National Electrical Contractors Association	—
OSHA	Occupational and Safety Health Administration	—
$P$	total radiant power	W
$P$	perimeter	m (ft)
$Q_\lambda$	spectral radiant energy	J/m
$r$	radius	m
$R$	ratio	—
RCR	room cavity ratio	—
$s$	distance	m
SC	sky component	—
SPD	standard illuminants or spectral power distributions	—
$t$	time	s
$T$	absolute temperature	K
$T$	transmittance	—
UV	Ultra Violet	—
$U_\lambda$	blackbody radiant energy	J/m
$v$	velocity	m/s
$W$	width	m (ft)
WCR	well cavity ratio	—

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### **Symbols**

$a$	spectral absorption factor	—
$e$	spectral emissivity	—
$h$	efficiency	—
$q$	angle	rad
$\lambda$	wavelength	m
$\nu$	frequency	Hz
$r$	spectral reflection factor	—
$S$	Stefan-Boltzmann constant	$5.670 \times 10^{-8} \text{ W}/(\text{m}^2 \times \text{K}^4)$
$S_L$	luminance (or radiance) Stefan-Boltzmann constant	$1.804 \times 10^{-8} \text{ W}/(\text{m}^2 \times \text{sr} \times \text{K}^4)$
$t$	net transmittance	—
$\tau$	spectral transmission factor	—
$F$	luminous flux	lm
$\omega$	solid angle	sr

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### **Subscripts**

<i>0</i>	initial (zero value)
<i>a</i>	absorbed
avg	average
<i>c</i>	controls or critical
<i>c-wp</i>	ceiling to workplane
<i>CC</i>	ceiling cavity
<i>d</i>	diffuse
<i>D</i>	direct
<i>e</i>	excitance, that is, radiometric, or radiant
<i>f</i>	final / frequency
<i>f</i>	per unit of frequency
<i>g</i>	gap
<i>i</i>	incident
<i>ic</i>	incident critical
in	inches
<i>L</i>	luminance or radiance
max	maximum
<i>r</i>	reflected
<i>RC</i>	room cavity
<i>s</i>	skylights or surface
<i>t</i>	total
<i>v</i>	vision, that is, photometric or luminous
<i>w</i>	wall, well, or workplane
<i>xh</i>	exterior horizontal
<i>/</i>	per unit wavelength

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### **HISTORY AND OVERVIEW<sup>2</sup>**

One of the earliest known theories of light was that it was emitted by one's eyes. Once Aristotle pointed out that we should, thus, be able to see in the dark but cannot, this theory lost standing. Sir Isaac Newton advanced the corpuscular theory in the 17th and 18<sup>th</sup> centuries from the observation that moving particles, or corpuscles, possess kinetic energy. His theory noted that light was radiated from luminous bodies, traveled in straight lines, and acts on the retina to provide a visual sensation.

Simultaneous with Newton's particle theory of light was Christiaan Huygens wave theory of light. His theory noted that light is a product of the molecular vibration of luminous material, is transmitted through an ether (now considered non-existent), and acts on the retina. The two theories fought for supremacy until the 19<sup>th</sup> century when James Clerk Maxwell developed his theory of electromagnetic radiation, which seemed to settle the case for light's being a wave. His theory noted that light was emitted from luminous material as radiant energy in the form of an electromagnetic wave, which then acts on the retina.

In the 20th century Max Planck developed quantum theory, which again reasserted that light was a particle. His theory noted that energy is emitted in discrete quantities known as photons, the magnitude of which is determined by the product of Planck's constant,  $h$ , and the frequency of the photon,  $\nu$  (see Eq. 6). The particle and wave theories were unified through the efforts of Louis de Broglie and Werner Heisenberg in what is currently termed the wave-particle duality. Their theory noted that every object of mass has an associated wavelength given by Eq. 1 (called the de Broglie Wavelength).

#### **Equation 1: Wavelength**

$$\lambda = \frac{h}{mv}$$

They further asserted one cannot determine if light is a wave or a particle, that indeed, it is both.

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<sup>2</sup> The material in this course is developed from the author's *Electrical Engineering Reference Manual* [A] illumination chapter, which was updated to include lighting design methods in Ref. [B]. Further, details, and an excellent source for further study, is found in Ref. [C]. References in the text will be shown in the "[\*]" format.

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For the purposes of lighting design, the quantum and electromagnetic theories provide the necessary theoretical background. Light is radiant energy capable of exciting the retina and producing a visual sensation. The efficiency of systems in radiating energy and the effects of that light on our visual sensations are the focus of the illumination engineer.

The International Commission on Illumination (CIE for Commission Internationale de l'Eclairage) was founded in 1913 as an information exchange for the science of lighting. The CIE was recognized by ISO (International Organization for Standards) as the standardizing body for “fundamental aspects of metrology, evaluation, and application of light and color including other radiation aspects in the optical radiation range.” ANSI, as part of the ISO, utilizes and disseminates standards. IEEE, NFPA, NEC, and many other bodies issue illumination (commonly called lighting) requirements.<sup>3</sup> The NEC (National Electrical Code), for example, specifies illumination requirements for health care facilities in NEC Art. 517. The NEC does not, however, specify the actual lighting level, leaving that for the (National Fire Protection Association) NFPA 101 Light Safety Code. Nevertheless, there is no description on how to attain the NFPA 101 Code-specified levels in NEC or the NFPA 101. Definitive guidance may be found for illumination design (that is, calculation of the light levels required while accounting for a variety of secondary effects) in the Illuminating Engineering Society of North America's IESNA Lighting Handbook: Reference & Application. Chapter 10 of the Lighting Handbook contains the IESNA Lighting Design Guide, which should be the starting point for any illumination design.

The information in the present course covers light (electromagnetic wave) principles; general knowledge of lighting—units, terminology, interaction with matter; optical control; and design methods and considerations.

Both the SI and the customary U.S. systems are used in illumination engineering. Unless otherwise indicated (usually with the U.S. units in parenthesis), the SI system is used in this chapter.

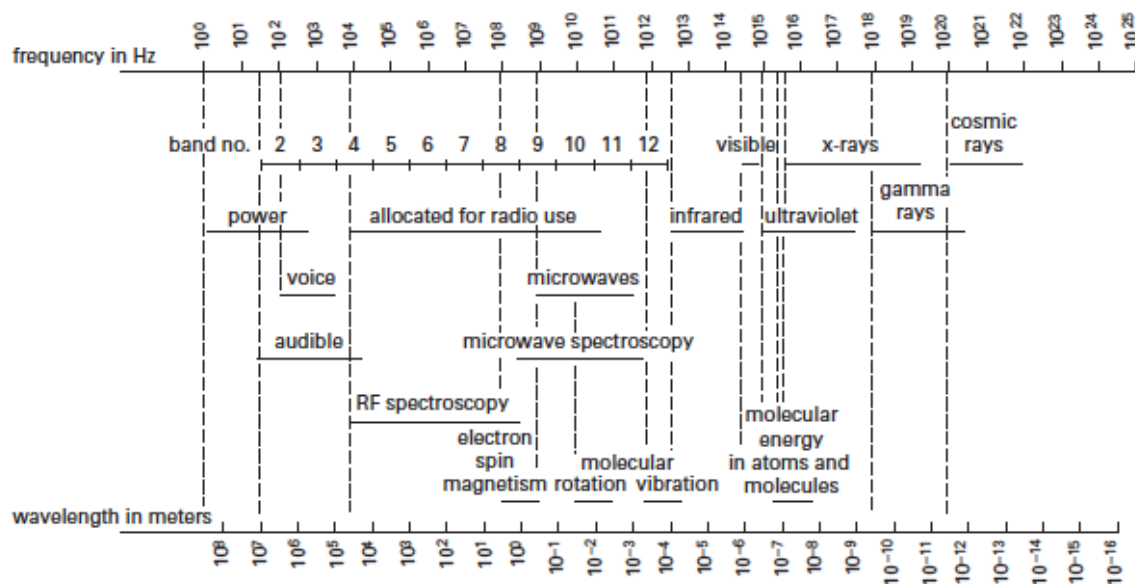
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<sup>3</sup> Lighting standards are numerous and can be found in Ref. [D]. Code impacts on lighting are in Ref. [E]. OSHA, NECA, and the DOE, among others also have standards for their areas of concern.

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### Electromagnetic Waves<sup>4</sup>

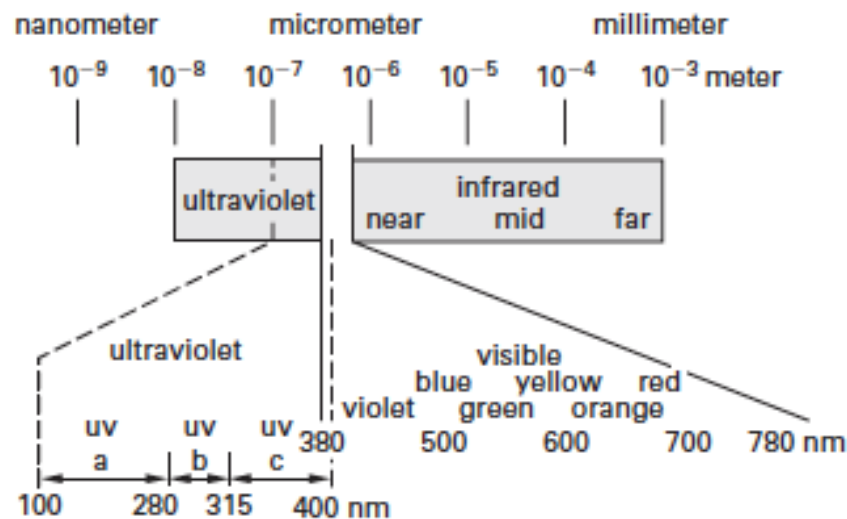
According to electromagnetic wave theory, *accelerating* electric charges radiate energy in the form of electromagnetic waves. This radiant energy is represented in a spectrum according to wavelength (or frequency)—see Fig. 1. The radiant energy portion of the spectrum extends from wavelengths of  $10^{-16}$  m to approximately  $10^5$  m. Wavelengths are normally given in meters, and frequency in hertz. Wavelengths, however, are often expressed in microns or micrometers ( $10^{-6}$  m), nanometers ( $10^{-9}$  m), and ångström [Å] ( $10^{-10}$  m). The nanometer is commonly used for the ultraviolet (UV) and visible regions, and the micrometer in the infrared (IR) region. Three regions, ultraviolet, visible, and infrared, comprise the primary electromagnetic spectrum area of concern for the illumination engineer. [F] These regions are shown in Fig. 2.



**Figure 1: Electromagnetic Spectrum**

<sup>4</sup> The topic of electromagnetic waves is often termed electromagnetic radiation. The term “radiation” has come to have negative implications, and so is often avoided. Nevertheless, there are numerous positive aspects to radiation. Indeed, radiation is necessary for human life to exist. The terms “electromagnetic wave” and “electromagnetic radiation” will be used interchangeably as the topic demands.

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**Figure 2: Ultraviolet, Visible, and Infrared Radiant Energy**

All electromagnetic waves (radiation) travel at the speed of light in a vacuum, approximately 186,000 mi/sec or  $3 \times 10^8$  m/s. The distance,  $s$ , traveled by the electromagnetic radiation, is given by  $s = \lambda \nu t$ , where  $\nu$  is the frequency of the wave,  $\lambda$  is the associated wavelength, and  $t$  is the time. Setting the time,  $t$ , equal to 1 s results in the distance traveled in that same time period, which corresponds to the speed of light,  $c$ . This results in Eq. 2, which is valid for all types of electromagnetic radiation.

### Equation 2: Speed of Light

$$c = \lambda \nu$$

The product of wavelength and frequency,  $\lambda \nu$ , is constant,  $c$ , in a vacuum. This effect, from Einstein's special theory of relativity, results in time dilation and length contractions that can be adjusted for with Lorentz transformations. Such effects result in a red shift (that is, the visible light from an object that is rapidly receding from an observer appears in a redder region of the frequency spectrum than it would if the object were stationary, and the light from a rapidly approaching object appears bluer). The frequency shift is called the Doppler shift or Doppler effect. The Lorentz factor,  $\gamma$ , is represented in Eq. 3, where the common relativity symbols,  $\gamma$  and  $\beta$ , are used.

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#### **Equation 3: Lorentz Factor**

$$g = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - b^2}}$$

When an electromagnetic wave passes through from one medium to another, the frequency remains constant while the velocity and wavelength change. In terms of wavelength, Eq. 2 is written as

#### **Equation 4: Medium Impacts**

$$\lambda = \frac{c}{U}$$

When the medium is not a vacuum in Eq. 4, the speed of light is replaced by the velocity,  $v$ , and the index of refraction,  $n$  (also called the refractive index), giving Eq. 5. (The index of refraction for a vacuum is equal to one, making the velocity equal to that of light,  $c$ , which brings Eq. 5 back to the form of Eq. 4.) The resulting change of speed (velocity) and wavelength bends the light (electromagnetic wave) toward the surface normal in a material denser than the original medium, and away from the normal in a less dense medium.

#### **Equation 5: Non-Vacuum Medium Impacts**

$$\lambda = \frac{vn}{U}$$

The frequency of the electromagnetic wave remains constant in any given medium because the energy contained in the wave remains constant, a result of the conservation of energy. Thus, the wavelength and velocity must change as the medium changes. The energy of a quantum or photon is given by Eq. 6. The product  $h\nu$  in Eq. 6 indicates that the energy is constant and dependent only on the frequency. The second and third portions of Eq. 6 combine to give Eq. 4, while the third and fourth portions combine to give Eq. 5. The net result is that light, an electromagnetic wave, retains its energy (and therefore, its frequency) as it passes through differing media. However, the light must change speed (velocity) and wavelength to do so, thereby changing direction. (Or, in terms of optics, the light ray—a beam of light with a small cross section—is refracted.)

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**Equation 6: Light Energy**

$$E = h\mathcal{U} = \frac{hc}{\lambda} = \frac{h\mathcal{U}n}{\lambda}$$

The speeds of light (that is, electromagnetic waves) in several substances of concern are given in Table 1. Note that the speed of an electromagnetic wave in air differs from that in a vacuum by less than 0.03%, and thus the two mediums are often considered equivalent for design purposes.

**Table 1: Speed of Light (Sodium D-lines at 589 nm) in Various Mediums**

medium	speed (m/s)
vacuum	$2.9979 \times 10^8$
air (760 mm Hg at 0°C)	$2.9972 \times 10^8$
water	$2.2492 \times 10^8$
crown (optical) glass	$1.9822 \times 10^8$

**Electromagnetic Spectrum: Ultraviolet**

The UV spectrum division shown in Fig. 2 is that given by the CIE. Ultraviolet light, though primarily not visible, has potential significant biological effects, as indicated in Table 2.

**Table 2: Ultraviolet Light: Biological Division**

wavelength (nm)	region effect	comments
180–220	ozone production	decomposes O <sub>2</sub> to O <sub>3</sub>
220–300	bactericidal (germicidal)	destroys bacteria, fungi, and viruses
280–320	erythema	reddening of the skin
300–400	black light	used for rating the effectiveness of lamps on fluorescent materials (excluding phosphors)

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As Table 2 indicates, UV light can have biological (physiological) effects, some of which are not desirable. By limiting lighting systems emissions to the black light region (300–400 nm), the negative biological effects of UV can be avoided. It should be noted that positive effects also exist. UV light is used by the body to generate vitamin D and to create the protective pigmentation that minimizes the negative effects.

### **Electromagnetic Spectrum: Visible Light**

The visible portion of the spectrum shown in Fig. 2 is that given by the CIE. The range is somewhat variable, but for practical purposes it is from  $3.8 \times 10^{-7}$  m to  $7.8 \times 10^{-7}$  m (380 nm to 780 nm). This region comprises many colors. Visible light of “many colors” is called polychromatic light to indicate that such light contains multiple wavelengths. When wavelengths of all or nearly all of the colors are included, the light is called white light. Thus, white light is not a color, but a combination of colors. The human retina is most sensitive to light at the center of the visible spectrum,  $5.55 \times 10^{-7}$  m, or 555 nm. This wavelength is the basis of the unit for luminous intensity, the candela. The visual sensation from differing wavelengths in the visible spectrum is sensed as color. The approximate color sensations are given in Table 3.

**Table 3: Color Versus Wavelength**

color	wavelength (nm)
violet	380–450
blue	450–495
green	495–570
yellow	570–590
orange	590–620
red	620–750

As Table 3 indicates, visible light is comprised of numerous colors. Numerous colors of light, including white light, have biological effects. Such light can result in burns, alter the effectiveness of drugs (drug photosensitivity), form lesions, and cause numerous other negative effects. The positive effects include altering one’s mood, minimizing jet lag, regulating biological rhythms, hormonal activity, and others.

The principal source of natural light is the sun, specifically the photosphere, a plasma envelope that radiates energy in the visible band. The sun’s illuminance of the Earth varies with the time of



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day and year, weather, and other factors. The CIE specifies *standard illuminants* or *spectral power distributions* (SPDs) for use in computing colors, design, and comparison applications.<sup>5</sup> The preferred standard illuminate for the sun is D<sub>65</sub>, which is light at a correlated color temperature of 6500 K. The *correlated color temperature* (CCT) for a source is the absolute temperature of a blackbody whose chromaticity most closely resembles that of the light source. The chromaticity ( $x, y, z$ ) values for the Sun standard D<sub>65</sub> are 0.33, 0.66, 0.01.<sup>6</sup> The  $x$  value correlates with red, the  $y$  value with green, and the  $z$  value with blue. (Hence the term RGB when referring to color reference systems.) The human eye has short, medium, and long wavelength sensors (blue, green, and red, respectively). The primarily red and green mixture of the Sun's chromaticity results in the visual sensation of a yellow Sun. [G]

Other natural sources of light include *sky lighting*, caused by the Rayleigh scattering of incoming electromagnetic radiation from the sun; fire; moonlight; lightning; the Aurora Borealis (northern lights) and Aurora Australis (southern lights); and bioluminescence, which is a form of chemiluminescence produced by plants in which light is produced by plant and animals through the process of oxidation.

### Electromagnet Spectrum: Infrared

The infrared (IR) portion of the spectrum shown in Fig. 2 extends from  $0.78 \times 10^{-6}$  m (0.78  $\mu$ m) to  $1.0 \times 10^{-3}$  m (1000  $\mu$ m). The IR spectrum is arbitrarily divided as shown in Table 4.

**Table 4: Infrared Band Versus Wavelength**

IR band	wavelength ( $\mu$ m)
near (short wavelength)	0.78–1.4
mid (medium wavelength)	1.4–3.0
far (long wavelength)	3.0–1000

<sup>5</sup> Different SPDs can result in the same color. Such SPDs are called metamers.

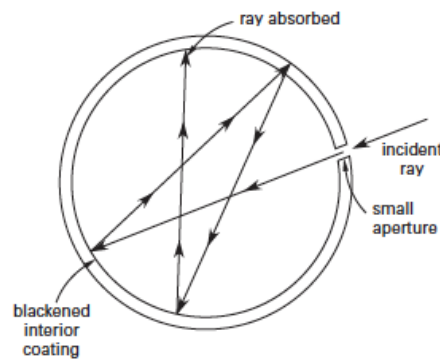
<sup>6</sup> Chromaticity values  $x, y, z$  are the ratios of the tristimulus values  $X, Y, Z$  to their sum. That is  $x = X / (X + Y + Z)$ , and so forth. The chromaticity values are usual specified in a graph of  $x$  and  $y$ , only given that  $x + y + z = 1$ . The tristimulus values define all metametric pairs (SPDs) by providing the amount of each primary color, (red, green, blue or  $X, Y, Z$ ) required by a standard observer to match the color being specified. (Think of that next time you're in the paint store asking for your favorite color.)

## *Illumination*

The near IR region results in the same biological effects as visible light. The mid and far IR regions can cause burns, generate cataracts, cause erythema (a different form than that from UV radiation), and other negative effects. The positive aspect is that IR radiation is useful for radiant heating.<sup>7</sup>

### **Blackbody Radiation**

A blackbody is an ideal body that would absorb all incident electromagnetic radiation. A blackbody is also known as a hohlraum or an ideal radiator. The term “blackbody” arises because there is no transmission or reflection of incident electromagnetic radiation. (Additionally, below approximately 1000 K any visible light incident upon a blackbody is absorbed, and thus the body appears black.) An approximation of an ideal blackbody created in the laboratory is shown in Fig. 3. A small aperture ensures any incident radiation is likely to be absorbed before it can escape, thus simulating blackbody characteristics.<sup>8</sup>



**Figure 3: Laboratory Blackbody**

While a blackbody neither transmits nor reflects incoming electromagnetic radiation, it does absorb such radiation and emit thermal radiation. Thermal radiation is the energy radiated by

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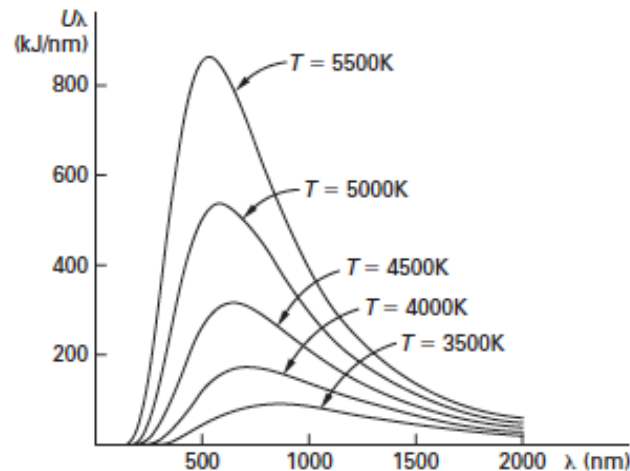
<sup>7</sup> Radiant heating is the source of warmth one feels on a sunny day, or next to a campfire on a cold night, or next to a hot substance, such as steel coming out of a furnace. Unlike the engineering of both UV and visible energy, design with IR energy is usually more concerned with the total energy deposited on a surface rather than the wavelength. Common applications of IR energy include industrial heating, drying or curing, baking in an oven, and photo-reproduction. However, the wavelengths or spectral characteristics of IR are important in IR detectors and viewing devices.

<sup>8</sup> An interior coating of lampblack results in an absorption of approximately 97% of incident light. Polished metal surfaces absorb less than 6% of the incident light. Most surfaces are between these extremes. (Lampblack is soot formed by burning oil, coal tar, resin, or other carbonaceous substances in an insufficient supply of air. It is essentially pure carbon.)

### Illumination

solids, liquids, and gases in the form of electromagnetic waves as a result of their temperature. The thermal radiation is in the UV, visible, and IR regions of the electromagnetic spectrum.

By maintaining the walls of the laboratory blackbody of Fig. 3 at a constant temperature, and measuring the radiation emitted from the small aperture, thermal radiation curves such as those shown in Fig. 4 are obtained.<sup>9</sup> Kirchhoff's law of thermal radiation implies that the shape of the curves is dependent on the temperature only, and not the composition or material of the body. Therefore, an object's temperature may be determined from the color of its emitted thermal radiation.<sup>10</sup>



**Figure 4: Radiated Power Versus Wavelength as Temperature Varies**

The concept of a blackbody is useful in the study of radiation phenomena. In the case of illumination engineering, by specifying both the magnitude of the radiation at any given wavelength and the absolute temperature (see Fig. 4 or 5), the characteristics (the spectral power density) of the light source are determined.<sup>11</sup>

<sup>9</sup> When the curves shown in Fig. 4 relate the power radiated by an actual object, the symbol used is  $Q$  or  $Q_\lambda$ . When the radiated power refers to that from a blackbody, the symbol used is  $U$  or  $U_\lambda$ .

<sup>10</sup> When an object is red hot, it is emitting radiation in the long wavelength portion of the visible band. As temperature increases, the glow changes color from red to blue. When the frequency is in the middle of the visible band, the object appears white, hence the term white hot.

<sup>11</sup> Such a specification is also reasonably accurate in the visible region of the spectrum for tungsten filaments and other incandescent sources of light. The temperature used, however, is the color temperature, which is the temperature of a blackbody that produces the same color as the specified radiator (light source).

## Illumination

Thermal radiation is exchanged according to Eq. 7. The symbol  $\alpha$  is the spectral absorption factor;  $\rho$  is the spectral reflection factor; and  $\tau$  is the spectral transmission factor. All these elements are dependent on the wavelength,  $\lambda$ .

### Equation 7: Thermal Radiation Exchange

$$a + r + t = 1$$

Kirchhoff's law of thermal radiation specifies that, at thermal equilibrium, the emissivity,  $\epsilon$ , of a body (or surface) equals its spectral absorption,  $\alpha$ . An object is a blackbody if for all frequencies Eq. 8 holds. (This, consistent with the definition of a black body, makes  $\rho = \tau = 0$ .)

### Equation 8: Blackbody Equation

$$e = a = 1$$

A blackbody, for a given area, radiates more total power at a given wavelength than any other light source at the same temperature.<sup>12</sup>

## Planck Radiation Law

The *Planck radiation law* is an expression providing the spectral radiance,  $L$ , of a blackbody as a function of wavelength and temperature. This is also known as *Planck distribution law* or *Planck's law*. It is Planck's law that provided the shape of the curves in Fig. 4. In terms of wavelength, a form of the law is given in Eq. 9. In terms of frequency, a form of the law is given in Eq. 10. The two equations have different units. Equation 42.9 is the radiance per unit wavelength interval and Eq. 10 is radiance per unit frequency interval. The two equations are related by Eq. 11.

### Equation 9: Spectral Radiance (Wavelength)

$$L(\lambda, T) = L_\lambda = \left( \frac{2hc^2}{\lambda^5} \right) \left( \frac{1}{e^{hc/\lambda kT} - 1} \right)$$

---

<sup>12</sup> The unit of luminous intensity was, until 1979, based on a blackbody operated at 2042 K (the freezing point for platinum). It was 60 cd/m<sup>2</sup>. The candela is now defined as the luminous intensity of a 555.016 nm source with a radiant intensity of (1/683) W/sr. The candela is thus defined in terms of an electrical unit, the watt, which is much easier to measure, and not dependent upon the international temperature scale, which has changed over time.

## *Illumination*

### **Equation 10: Spectral Radiance (Frequency)**

$$L(f, T) = L_f = \left( \frac{2hf^3}{c^2} \right) \left( \frac{1}{e^{hf/kT} - 1} \right)$$

### **Equation 11: Planck Radiation Law Equivalence**

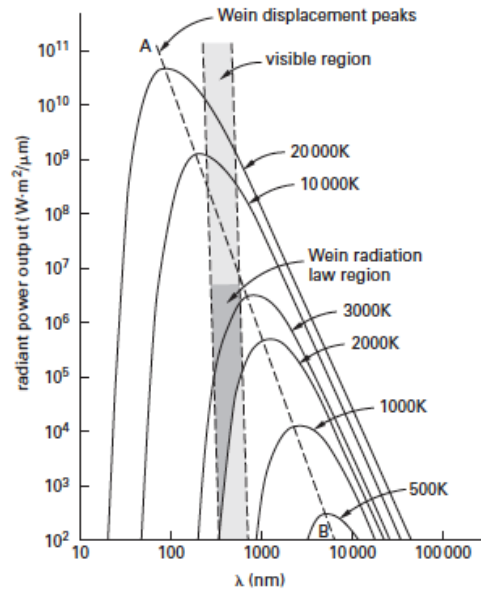
$$L(\lambda, T)d\lambda = L(f, T)df$$

In the temperature range of incandescent filament lights (2000 K to 3400 K), with a wavelength in the visible range (380 nm to 780 nm), a simplification of the Planck radiation law (specifically, Eq. 9) can be used as a useful approximation (within 1%) of the spectral radiance. This is called the *Wein radiation law*. The Wein radiation law, given by Eq. 12, is applicable to the shaded region in Fig. 5 between 2000 K and 3400 K.

### **Equation 12: Wein Displacement Law**

$$L(\lambda, T) = L_\lambda = \left( \frac{2hc^2}{\lambda^5} \right) e^{-\left( hc/\lambda kT \right)}$$

## Illumination



**Figure 5: Blackbody Radiation Curves [H]**

### Wein Displacement Law

In Fig. 5, as the absolute temperature increases, the peak of the radiated power curve shifts to the left toward higher frequencies and shorter wavelengths. The law itself is similar to the Planck radiation law, but two principle corollaries to the law are the focus for the illumination engineer. The first gives the maximum wavelength for a given temperature, Eq. 13. (The term  $b$  in Eq. 13 is a constant whose current value is  $2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$ .) Equation 14 gives the maximum spectral radiance for a given wavelength. (The term  $b$  in Eq. 14 is a constant whose current value is  $4.0956 \times 10^{-4} \text{ J/s} \cdot \text{sr} \cdot \text{m}^2 \cdot \text{m} \cdot \text{K}^5$  or  $\text{W/sr} \cdot \text{m}^2 \cdot \text{m} \cdot \text{K}^5$ .)<sup>13</sup>

### Equation 13: Peak Wavelength given Temperature

$$l_{\max} = \frac{b}{T}$$

<sup>13</sup> The units are shown with an  $\text{m}^2$  and  $\text{m}$  separate instead of  $\text{m}^3$ . This is to emphasize that the radiance term ( $L$ ) (with units of  $\text{W/sr} \cdot \text{m}^2$ ) is here given as spectral, that is per unit wavelength (hence units of  $1/\text{m}$ ). This also applies to  $L_i$  and  $L_f$  in Eqs. 9 and 10. Units are shown in a similar fashion in the nomenclature. It should be noted that the radiance,  $L$ , is dependent upon the direction of light that strikes a surface. That is, a  $dA \cos \theta$  term is part of the definition. For information on units see Appendix A.

## *Illumination*

### **Equation 14: Maximum Spectral Radiance given Temperature**

$$L_{\max} = bT^5$$

The peaks calculated from Eqs. 13 and 14 are located on the dashed line between points A and B in Fig. 5 (once the results are adjusted for wavelengths in micrometers as plotted in the figure).

### **Stefan-Boltzmann Law**

The *Stefan-Boltzmann law* stipulates that the total radiant power per unit area of a blackbody varies as the fourth power of the absolute temperature, as given in Eq. 15. This is also known as the *fourth-power law* or *Stefan's law*.

### **Equation 15: Stefan's Law**

$$\frac{P}{A} = ST^4$$

In terms of the *luminous exitance*,  $M$ , the law is written

### **Equation 16: Luminous Exitance**

$$M_u = ST^4$$

In terms of the *radiance*,  $L$ , the law is written

### **Equation 17: Radiance Exitance**

$$L_e = S_L T^4$$

Stefan-Boltzmann's law applies to the entire spectrum, that is, it measures the total radiated power, not just that in the visible portion of the spectrum.

## Illumination

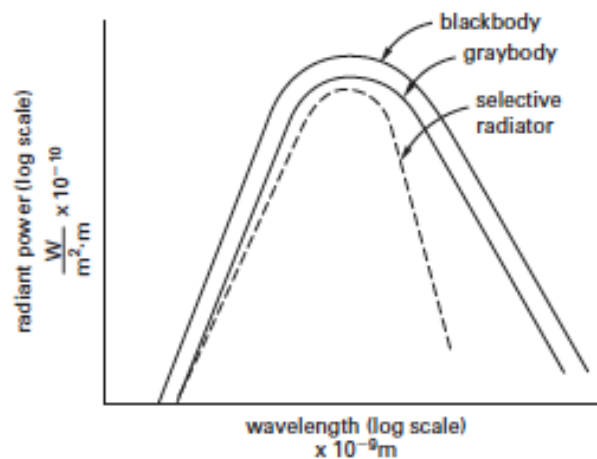
### Graybody and Selective Radiators

The blackbody radiator has the most emissive power known. Other radiators must be corrected or compared to a blackbody radiator at the same temperature. The correction factor is called the *spectral emissivity*,  $\epsilon$ , which is a function of the wavelength,  $\epsilon(\lambda)$ . The spectral emissivity is the ratio of the radiation emitted by a surface at a specified wavelength to the radiation emitted by an ideal blackbody radiator at the same wavelength and temperature. In terms of the luminous (in this case, radiant) exitance, the ratio is

#### Equation 18: Spectral Emissivity

$$\epsilon = \frac{M_e}{M_{\text{blackbody}}}$$

A graybody is an energy radiator that has a constant emissivity. That is, a graybody has a blackbody energy distribution reduced by the constant factor,  $\epsilon$ , at all wavelengths. A *selective radiator* is a radiator with an emissivity that varies with wavelength. All three types of radiators are shown in Fig. 6.



**Figure 6: Radiator Types**



## *Illumination*

### **Color Temperature**

The radiation characteristics of a blackbody may be determined from Planck's law by specifying only two parameters, the magnitude of the radiation at a given wavelength ( $L$ ) and the absolute temperature ( $T$ ). The same may be done in the visible region of the spectrum for selective radiators, such as the tungsten filaments in many incandescent sources, by using a temperature other than the actual temperature of the filament. This temperature is the *color temperature*. The color temperature is the temperature of a blackbody radiator of essentially the same color. Of note, the *distribution temperature* is the temperature of a blackbody whose relative spectral power distribution is most nearly that of the selective radiator.<sup>14</sup>

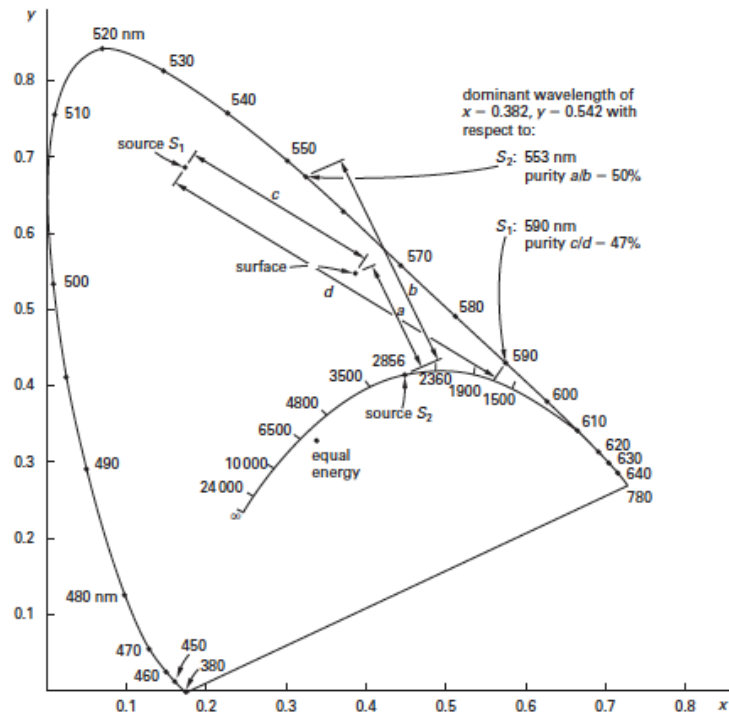
The color temperature is calculated from the chromaticity coordinates. An example of such a calculation using the (x, y) chromaticity coordinates is shown in Fig. 7. The details of the calculation are in Ch. 4 of the IESNA Lighting Handbook, and in the CIE standards. Once the wavelength is determined, the color temperature is known. Distances in the CIE (x, y) diagram (or X, Y, Z space) did not correlate well with perceived differences in color. Therefore, a CIE Uniform Color Space (UCS) was developed. This is shown in Fig. 8. The  $u'$  and  $v'$  axes correspond to values determined by formulas for adjusting X, Y, Z space to the updated coordinate system used in the CIE Uniform Color Space (UCS).

The color temperature and distribution temperature specify the output of incandescent (hot body) sources only. For other sources, the correlated color temperature (CCT) is used. The CCT and the color temperature both are compared to a blackbody whose chromaticity most nearly matches that of the source. The terms simply differentiate the source. Both are commonly referred to as the color temperature.

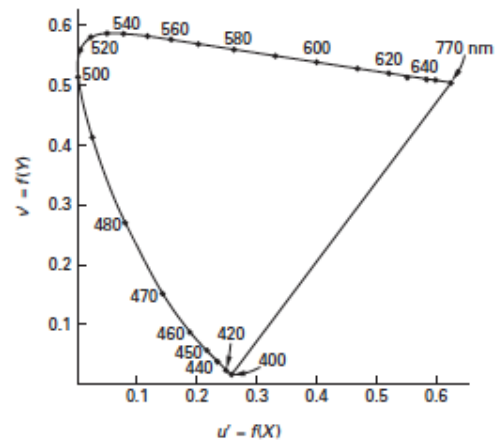
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<sup>14</sup> Many different spectral power distributions (SPDs) exhibit the same color. The distribution temperature determines the specific power at a given wavelength, which is important in determining the overall effects of the light, including the physiological effects.

### *Illumination*



**Figure 7: CIE Chromaticity Diagram, 1951 / Dominant Wavelength and Purity**  
[Source: Reference D]



**Figure 8: CIE Uniform Color Space (UCS), 1976**  
[Source: Reference D]

## *Illumination*

### **Atomic Structure: Electromagnetic Radiation**

Electrons in orbit around the nucleus of an atom exist in various quantized energy states, or levels, that are categorized as groups called shells. Those electrons in the inner shells are not easily removed. Those in the outer shells are readily excited to higher energy levels and can also be removed from their associated atoms. These outer shell electrons are called valence electrons. When UV or visible radiation is absorbed by an outer shell electron, the electron moves to a higher energy state. (In photodiodes, this higher energy state is in the conduction band.) When the electron eventually returns to its original energy level, it emits a photon of radiation with energy given by

#### **Equation 19: Transition Energy**

$$E_2 - E_1 = h\nu_{21}$$

The energy level differences can be represented as a potential difference in volts. Equation 19 can be put in the useful form of Eq. 20, in which the wavelength in nanometers is given in terms of the potential difference, in volts. In terms of the energy gap, the wavelength is given by Eq. 21 and is illustrated in Fig. 9.

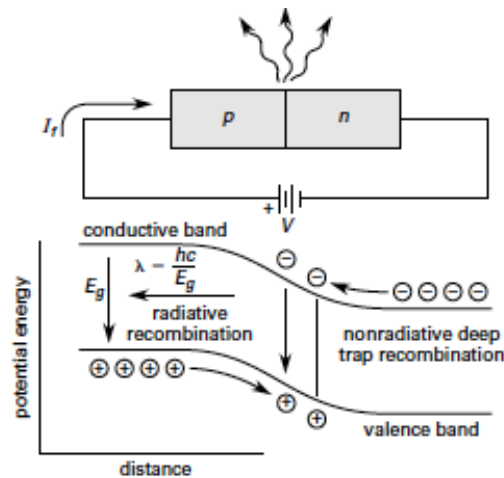
#### **Equation 20: Energy Gap: Voltage**

$$\lambda_{\text{nm}} = \frac{1239.76 \text{ nm} \cdot \text{V}}{DV}$$

#### **Equation 21: Energy Gap: Energy**

$$\lambda = \frac{hc}{E_g}$$

### Illumination

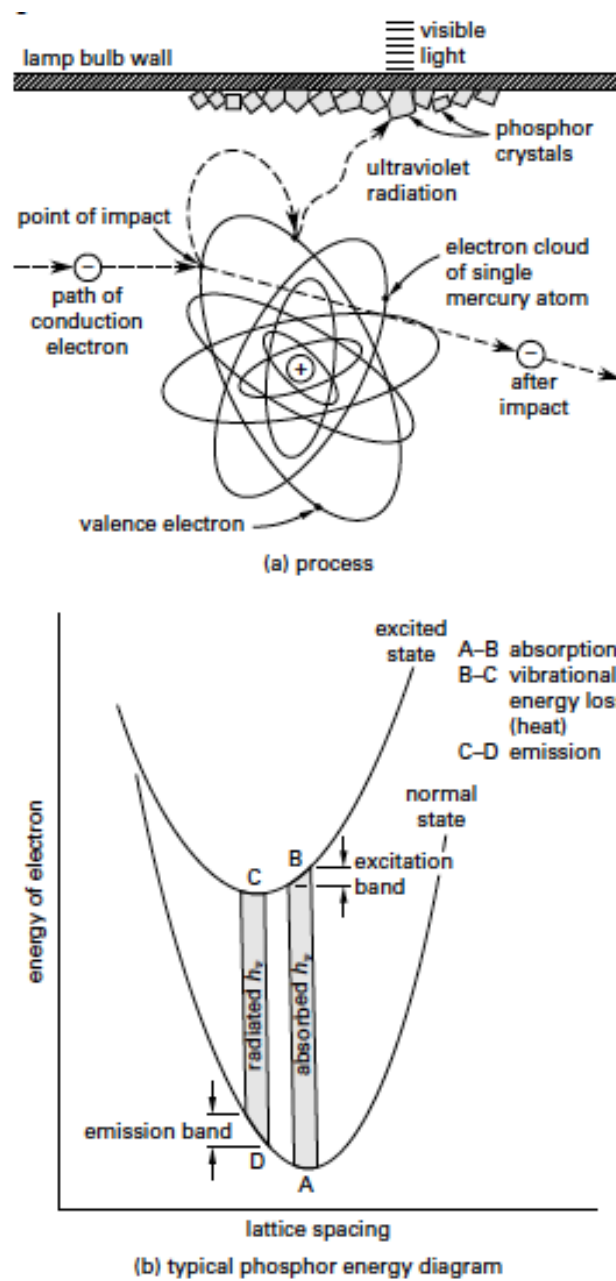


**Figure 9: Light Emitting Diode (LED)  $p$ - $n$  Junction [I]**

The cause of light emission is the electronic transitions from higher to lower energy states with the release of electromagnetic radiation (waves) to maintain the energy balance.<sup>15</sup> In spite of this, light sources are historically divided into two types, incandescent and luminescent. *Incandescent* light sources emit visible light because of their temperature. Examples are filament lamps, pyroluminescence (flames), candoluminescence (gas mantles), and carbon arc radiation. *Luminescent* light sources emit visible light caused by any factor other than their temperatures, including the following examples. Gas discharges and fluorescence, which are forms of immediate light release, are termed photoluminescence (see Fig. 10). Phosphorescence is a delayed release of light. Lasers emit coherent light. Electroluminescence includes lamps that may be AC capacitive or light emitting diodes. Electron excitation causes cathodoluminescence. Galvanoluminescence and chemiluminescence result from chemical changes; crystalloluminescence results from crystallization; thermoluminescence from heat; triboluminescence from friction; sonoluminescence from ultrasonics; and radioluminescence from the interactions of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and X-ray particles with matter.

<sup>15</sup> Care should be taken in the study of illumination as the terms energy and power are often used interchangeably, which may lead to errors in calculations.

### Illumination



**Figure 10: Typical Phosphor Process and Energy**

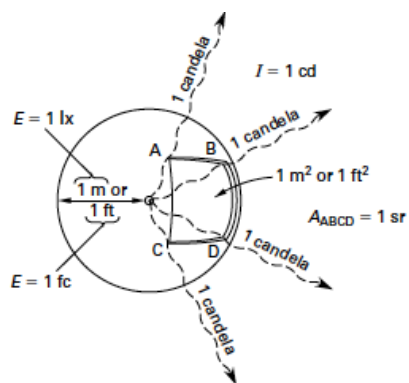
## Illumination

### The Candela and Lumen

The base SI unit for luminous intensity,  $I$ , is the candela (cd), which represents the luminous flux per unit solid angle, or steradian. As defined in the SI, the *candela* is the luminous intensity, in a given direction, of a source that emits a monochromatic radiation of frequency  $540 \times 10^{12}$  Hz, and that has a radiant intensity in that direction of  $1/683$  watt per steradian. The frequency in the definition correlates with a wavelength of 555 nm, which is the wavelength of maximum photopic (cone) or daylight efficiency for human vision.<sup>16</sup> Additionally, there is a total of 683 lm available per watt at 555 nm, though the maximum luminous efficacy (all radiation in the visible band) for an ideal white source provides only 220 lm/W.

The lumen is considered a derived SI unit for luminous flux. It is actually the core of the other units and the focus for illumination design. The lumen, lm, is a unit relating the radiant flux produced in watts to visually effective radiation (that is, light) for the standard human observer.

The relationships between candelas (luminous intensity), lumens (luminous flux), and the corresponding illuminance units of lux (SI flux per unit area) and footcandles (customary U.S. flux per unit area) are shown in Fig. 11. In the SI system, at 1 m, a luminous intensity of 1 cd illuminates 1 lx over an area of 1 sr. In the customary U.S. system of units, at 1 ft, a luminous intensity of 1 cd illuminates 1 fc over an area of 1 sr. A 1 cd source provides  $4\pi$  total lumens of light.

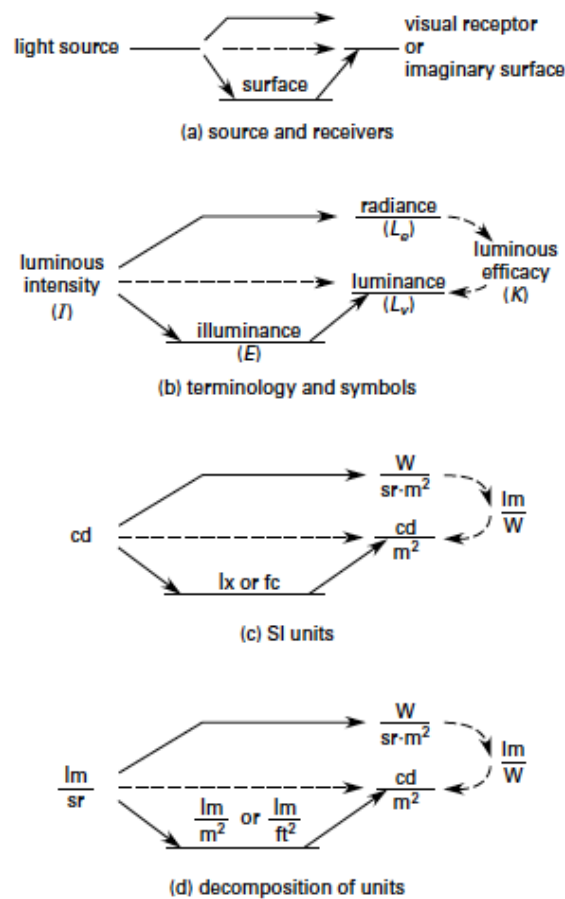


**Figure 11: Relationships Between Candela, Lumen, Lux, and Foot-Candle**

<sup>16</sup> There are two types of photoreceptors in the human eye. One is the cone or photopic receptor, responsible for daylight or bright vision. The other is the rod or scotopic receptor, responsible for nighttime or dark condition vision. The two types of receptor have different efficiencies at the same wavelength.

## Illumination

An aid in the study of illumination terms and relationships is shown in Fig. 12. The terms and units are discussed throughout this chapter. Additional information on luminance and illuminance units and conversion factors is given in the Appendix A and B.



**Figure 12: Illumination Terms and Relationships**

The luminous efficacy,  $K$ , is a measure of the luminous flux output emitted relative to the total source power input, and is given by Eq. 22. The term “luminous efficiency” has been used extensively for this term, therefore, the symbol  $\eta$  is used as well as  $K$ . Typical luminous efficacy values are shown in Table 5.

### Equation 22: Luminous Efficacy

$$K = \eta = \frac{F_u}{F_e} = \frac{F_u}{P}$$

## Illumination

**Table 5: Luminous Efficacy by Source**

power (W)	source	$K$ (lm/W)
	candle	0.1
25/100/1000	tungsten lamp	10/16/22
100/1000	mercury arc	35/65
400	mercury fluorescent lamp	58
1000	carbon arc	60
40	fluorescent lamp	80
400/1000	metal halide lamp	85/100
400/1000	high-pressure sodium lamp	125/130
180	low-pressure sodium lamp	180
	ideal white light	220

The radiant exitance,  $M_e$ , is the radiant flux leaving a surface (that is, the source of the flux) in units of  $\text{W/m}^2$ , and is given by Eq. 23.

**Equation 23: Radiant Excitance**

$$M_e = \frac{F_e}{A_{\text{source}}}$$

The luminous exitance,  $M_v$ , (formerly called the luminous emittance, and roughly referred to as the brightness) is the luminous flux per unit area of the source ( $\text{lm/m}^2$ ), given by Eq. 24.

**Equation 24: Luminous Exitance**

$$M_u = \frac{F_u}{A_{\text{source}}}$$

---

**Example 1**

A 100 W lamp has a luminous flux of 4400 lumens. What is the lamp's luminous efficiency?

*Solution*

From Eq. 22,



*Illumination*

$$\begin{aligned}
 K = h &= \frac{F_u}{P} \\
 &= \frac{4400 \text{ lm}}{100 \text{ W}} \\
 &= 44 \text{ lm/W}
 \end{aligned}$$


---

**Example 2**

A tungsten filament emitting 1600 lm has a surface area of 0.35 in<sup>2</sup> ( $2.26 \times 10^{-4} \text{ m}^2$ ). What is its brightness?

*SI Solution*

$$\begin{aligned}
 M_u &= \frac{F_u}{A_{\text{surface}}} \\
 &= \frac{1600 \text{ lm}}{2.26 \times 10^{-4} \text{ m}^2} \\
 &= 7.08 \times 10^6 \text{ lm/m}^2
 \end{aligned}$$

*Customary U.S. Solution*

$$\begin{aligned}
 M_u &= \frac{F_u}{A_{\text{surface}}} \\
 &= \frac{1600 \text{ lm}}{\left( \frac{0.35 \text{ in}^2}{144 \frac{\text{in}^2}{\text{ft}^2}} \right)} \\
 &= 6.58 \times 10^5 \text{ lm/ft}^2
 \end{aligned}$$


---

## Illumination

### Luminous Intensity

The *luminous intensity*,  $I$ , of a source is the flux emitted per unit solid angle from the point where the source radiated. The units are candela (cd), which are lumens per steradian (lm/sr).<sup>17</sup>

#### Equation 25: Luminous Intensity

$$I = \frac{dF}{d\omega}$$

Equation 26 gives the luminous intensity from a uniformly radiating point source a distance  $r$  from that source.

#### Equation 26: Point Source Luminous Intensity

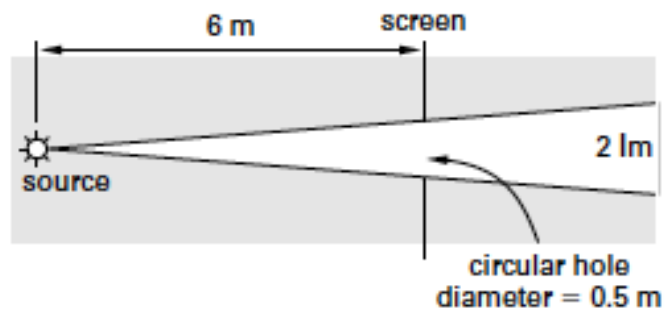
$$\begin{aligned} I &= \frac{F}{\omega} = \frac{F}{\left(\frac{A}{r^2}\right)} \\ &= \frac{F_t r^2}{4\rho r^2} \\ &= \frac{F_t}{4\rho} \end{aligned}$$

### Example 3

Two lumens pass through a circular hole (diameter is 0.5 m) in a screen located 6.0 m from an omnidirectional source. (a) What is the luminous intensity of the source? (b) What luminous flux is emitted by the source?

<sup>17</sup> The unit “steradian” is technically a ratio and is normally not shown. In photometry, and as approved by National Institute of Standards and Technology (NIST) for use in the SI system, the name “steradian” and the symbol “sr” are retained in expressions for units, for clarity.

### Illumination



### Solution

(a) The solid angle subtended by the hole is

$$\begin{aligned}
 \omega &= \frac{A}{r^2} \\
 &= \frac{\frac{\rho}{4} (0.5 \text{ m}^2)}{(6 \text{ m})^2} \\
 &= 0.005454 \text{ sr}
 \end{aligned}$$

From Eq. 26, the intensity is

$$\begin{aligned}
 I &= \frac{F}{\omega} \\
 &= \frac{2 \text{ lm}}{0.005454 \text{ sr}} \\
 &= 366.7 \text{ lm/sr}
 \end{aligned}$$

(b) From Eq. 26, the luminous flux is

$$\begin{aligned}
 F_t &= 4\rho I \\
 &= (4\rho \text{ sr}) \left( 366.7 \frac{\text{lm}}{\text{sr}} \right) \\
 &= 4608 \text{ lm}
 \end{aligned}$$

## Illumination

### Illuminance

The *illuminance*,  $E$ , is the areal density of the luminous flux at a point on a real or imaginary surface. The units are lux (lx), which are equivalent to lumens per square meter ( $\text{lm}/\text{m}^2$ ). (When referring to the radiant flux at a point on the surface, that is, the power incident to the surface, the units are watts per square meter,  $\text{W}/\text{m}^2$ , and the term *irradiance* is used. Subscripts are used to differentiate photometric from radiometric quantities only when necessary for clarification. The symbol changes from  $E$  to  $E_v$  for illuminance, and to  $E_e$  for irradiance. Additionally, the term illuminance is often called illumination, irradiance, and brightness.<sup>18</sup>

#### Equation 27: Illuminance

$$E = \frac{dF}{dA}$$

Equation 28 gives the illuminance from an omnidirectional source at a spherical receptor of radius  $r$ . (The receptor, as in all illumination equations, may be a real or imaginary area at which the illuminance is of interest.)

#### Equation 28: Omnidirectional Source Illuminance

$$E = \frac{F_t}{4\pi r^2}$$

Recommended illuminance (illumination) levels are specified either by the exact level or by category, based on the intended use of the space. The categories and recommended levels are shown in Table 6. Categories A, B, and C are for *orientation and simple visual tasks*, that is, tasks for which visual performance is of little importance. Categories D, E, and F are for *common visual tasks* for which visual performance is important. Categories D, E, and F represent the majority of commercial, industrial, and residential lighting requirements. Category G is for *special visual tasks* for which visual performance is critical, as in hospital operating rooms. Illumination levels in general should be within 10% of the indicated values.

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<sup>18</sup> Definitions using the SI standard are given throughout this text. Many terms, including irradiance, illumination, and brightness, are no longer recommended (that is, they are “deprecated”). However, they still occur in a variety of publications, and are used for clarity when speaking to those not familiar with illumination engineering. When in doubt, refer to ANSI/IES RP-16, *Nomenclature and Definitions for Illuminating Engineering*.

Illumination

Table 6: Recommended Illuminance

category	location/condition/ magnitude	lux	fc
A	public spaces	30	3
B	simple orientation/short visits	50	5
C	cooking space/simple visual tasks	100	10
D	visual task/high contrast required/large size	300	30
E	visual task/high contrast/small size	500	50
F	visual task/low contrast/ small size	1000	100
G	critical visual tasks	3000– 10 000	300– 1000

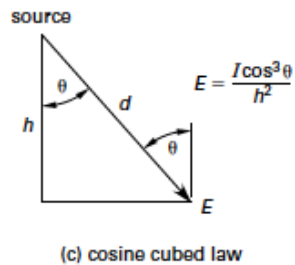
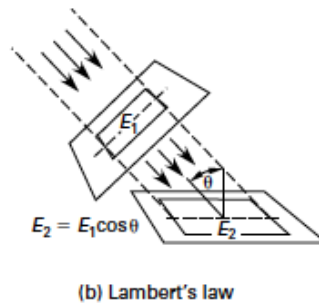
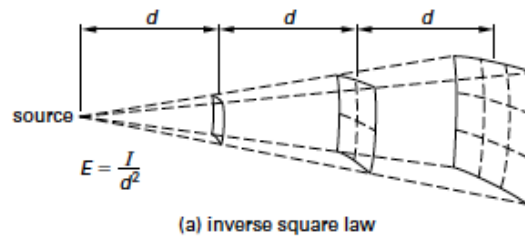
Illuminance: Inverse Square Law

The illuminance on a surface varies in accordance with the inverse square law, given by Eq. 29. The inverse square law is accurate to within 1% when the distance  $d$  is at least five times the maximum dimension of the source (or luminaire). This is called the *five-times rule*.

Equation 29: Inverse Square Law

$$E = \frac{I}{d^2}$$

### Illumination



**Figure 13: Illuminance Laws**

When comparing the illuminance (illumination) from the same source at two different points, the inverse square law takes the form of Eq. 30. The impact of the inverse square law on light flux is illustrated in Fig. 12(a).

#### Equation 30: Illuminance Comparison

$$E_1 r_1^2 = E_2 r_2^2$$

### *Illumination*

#### **Example 4**

A lamp radiating hemispherically and rated at 2000 lm is positioned 20 ft (6 m) above the ground. What is the illumination on a walkway directly below the lamp?

#### *SI Solution*

$$\begin{aligned}
 E &= \frac{F}{A} \\
 &= \frac{F}{\frac{1}{2} A_{\text{sphere}}} \\
 &= \frac{2000 \text{ lm}}{\left(\frac{1}{2}\right)(4\rho)(6 \text{ m})^2} \\
 &= 8.84 \text{ lx}
 \end{aligned}$$

#### *Customary U.S. Solution*

$$\begin{aligned}
 E &= \frac{F}{A} \\
 &= \frac{F}{\frac{1}{2} A_{\text{sphere}}} \\
 &= \frac{2000 \text{ lm}}{\left(\frac{1}{2}\right)(4\rho)(20 \text{ ft})^2} \\
 &= 0.796 \text{ lm/ft}^2 \quad (\text{fc})
 \end{aligned}$$

## Illumination

### Illuminance: Lambert's Law

The illuminance on a surface depends on the angle at which the light flux strikes the surface. *Lambert's law*, also called the *cosine law*, states that the illuminance on any surface varies with the cosine of the angle of incidence,  $\theta$ , as shown in Eq. 31 and in Fig. 12(b). When combined with the inverse square law, the result is Eq. 32.

#### Equation 31: Cosine Law

$$E_2 = E_1 \cos q$$

#### Equation 32: Cosine / Inverse Square Law Combination

$$E = \frac{I}{d^2} \cos q$$

### Illuminance: Cosine-Cubed Law

When determining the illuminance level at an angle from the source, a useful extension of Lambert's cosine law is the *cosine-cubed law*. This law is derived by considering the geometry of Fig. 12(c), which allows the substitution of  $h/\cos$  for  $d$  in the inverse square law, Eq. 29, resulting in

#### Equation 33: Cosine Cubed Law

$$E = \frac{I \cos^3 q}{h^2}$$

### Luminance

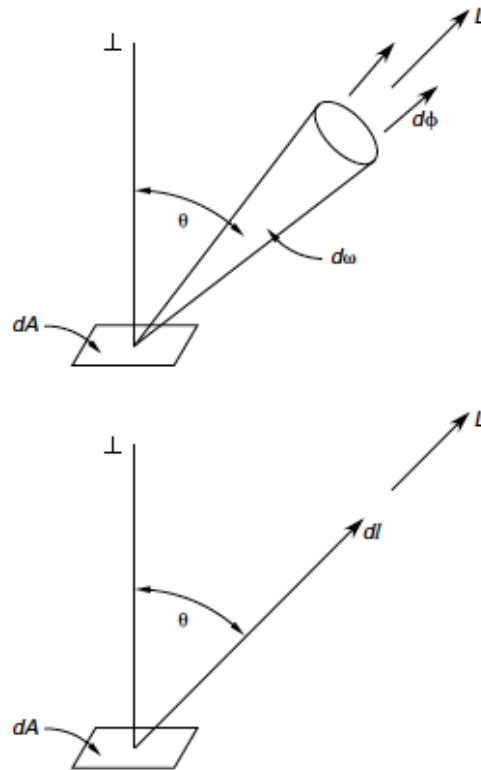
The *luminance*,  $L$ , is the ratio of the luminous intensity, in a given direction, of an infinitesimal element of a surface containing the point, to the orthogonally projected area of the element on a plane perpendicular to the given direction. Equation 34 represents the concept mathematically. Figure 13 illustrates the concept physically. (When referring to the radiant flux at a point on the surface, that is, the power incident to the surface, the units are watts per steradian per square meter,  $\text{W}/\text{sr} \cdot \text{m}^2$ , and the term *radiance* is used. Subscripts are used only to differentiate photometric from radiometric quantities, when necessary for clarification. The symbol changes from  $L$  to  $L_v$  for luminance, or to  $L_e$  for radiance.



## Illumination

### Equation 34: Luminous Mathematically

$$E = \frac{d^2\Phi}{dw dA \cos q} = \frac{dI}{dA \cos q}$$



**Figure 14: Mathematical Concept of Illuminance Illustrated**

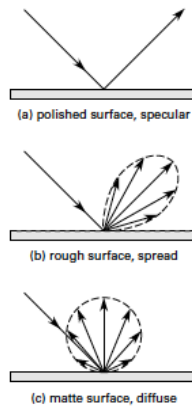
### Interaction of Light with Matter

Light travels through a vacuum as an electromagnetic wave. When the light makes contact with matter, some of the wave energy is absorbed by the matter, sometimes causing electrons to jump into higher energy states. (A polished metal surface, for example, will absorb only about 10 percent of the incident energy, reflecting the remaining 90 percent away.) Some of this absorbed energy is re-emitted when the electrons drop back to a lower energy level. Generally, the re-emitted light will not be at its original wavelength.

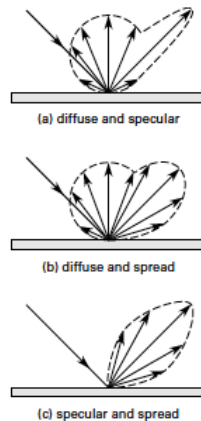
## Illumination

If the reflecting surface is smooth, the *reflection angle* for most of the light will be the same as the *incident angle* and the light is said to be *regularly reflected* (i.e., the case of *specular reflection*). If the surface is rough, however, the light will be scattered and reflected randomly (the case of *diffuse reflection*).

Examples of specular and diffuse reflections are shown in Fig. 15. Compound reflections are illustrated in Fig. 16.



**Figure 15: Reflections**



**Figure 16: Compound Reflections**

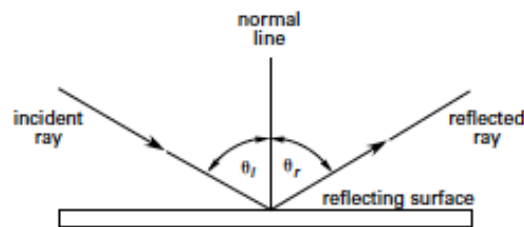
The energy that is absorbed is said to be *refracted*. In the case of an *opaque material*, the refracted energy is absorbed within a very thin layer and converted to heat. However, the light is able to

## Illumination

pass through a *transparent material* without being absorbed. Light is partially absorbed in a *translucent material*.

### Reflection

Reflected light leaves a reflecting surface at the same angle (as measured from a normal line) it approaches the surface (see Fig. 17).



**Figure 17: Reflections from a Surface**

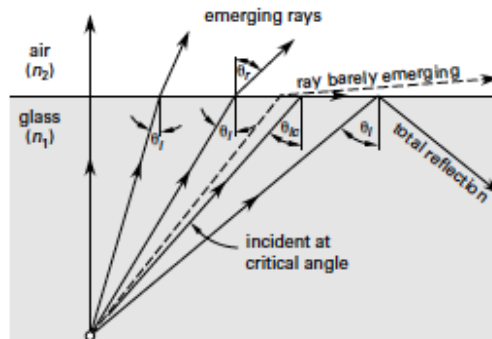
While reflections are normally associated with smooth opaque surfaces, light can also be totally reflected from a transparent surface if the incident angle is sufficiently large. Equation 35 gives the critical incident angle at which an optically transparent surface becomes totally reflecting. Total reflecting prisms are used in place of silvered mirrors when precise reflection is required. Total internal reflection is also the principle by which *optical fibers (light pipes)* transmit light.

### Equation 35: Critical Angle

$$\sin \theta_c = \frac{1}{n}$$

An example of the critical angle for a light ray interacting with glass (or a fiber-optic cable) is illustrated in Fig. 18. Control of the light radiation is vital to the output of a fiber-optic system. Therefore, fiber-optic cables are coated with a material with a very low refractive index,  $n$ , making the  $\sin \theta \approx 1$ .

### Illumination



**Figure 18: Critical Angle**

### Refraction

Refraction is the bending of light as it passes from one transparent medium into another. *Snell's law*, Eq. 36, relates the incident and refracted angles and predicts that the light will bend *toward the normal* when it enters an optically denser material. For a vacuum (and, for practical purposes, air),  $n_1 = 1$ .

#### Equation 36: Snell's Law

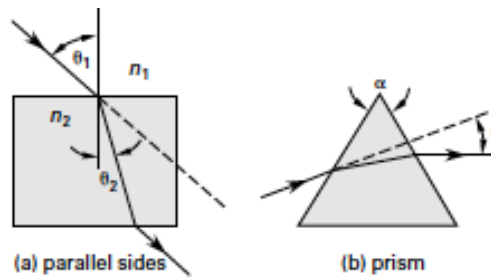
$$n_{\text{relative}} = \frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

If a light beam passes through a transparent medium with parallel surfaces, the emergent beam will be parallel to the incident beam, as illustrated in Fig. 19(a). However, the refraction due to a prism with apex angle  $\alpha$ , illustrated in Fig. 19(b), is more complex. Equation 37 predicts the minimum *angle of refraction* (*angle of deviation*) from the original path.<sup>19</sup>

---

<sup>19</sup> Different wavelengths (colors) will be reflected at slightly different angles.

### Illumination



**Figure 19: Refraction of Light**

### Equation 37: Angle of Refraction

$$n_{\text{relative}} = \frac{\sin \frac{a+q}{2}}{\sin \frac{a}{2}}$$

Refraction causes submerged objects to appear closer to the surface than they actually are. Equation 38 gives the apparent depth,  $D$ .

### Equation 38: Submerged Object Refraction

$$D_{\text{apparent}} = \frac{D_{\text{actual}}}{n_{\text{relative}}}$$

### Index of Refraction

While the speed of light in a vacuum is constant, it changes in different transparent media. The absolute *index of refraction* (refractive index),  $n$ , is the ratio of the speed of light in a vacuum (essentially the same as in air) to the speed of light in a particular medium.<sup>20</sup> The approximate absolute indices of refraction are listed in Table 7. It is not strictly constant but varies 1–2% over the visible light spectrum. This variation is disregarded, however, in simple studies.

---

<sup>20</sup> The **relative index of refraction**, the ratio of the speeds of light in two different media, is also encountered

## Illumination

### Equation 39: Index of Refraction

$$n = \frac{c_{\text{vacuum}}}{c_{\text{medium}}} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{c_{\text{medium, m/s}}}$$

**Table 7: Approximate Absolute Indices of Refraction**  
(at  $\lambda = 5.893 \times 10^{-7} \text{ m}$ )

medium	$n$
air (20°C, 1 atm)	1.00029
benzene	1.50
borosilicate crown glass	1.5243
diamond	2.417
flint glass, dense	1.6555
hydrogen (20°C, 1 atm)	1.00013
ice	1.31
quartz, fused	1.46
salt	1.53
water (20°C)	1.3330

Most transparent substances including glass, water, air, and polymethyl metacrylate (Lucite™) are isotropic; that is, light travels at the same speed in all directions. However, some transparent crystals are *anisotropic*.

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### Example 5

At a particular frequency corresponding to red light, the refractive index of water is approximately 1.3300. What is the speed of this light in water?

#### *Solution*

From Eq. 39,

### Illumination

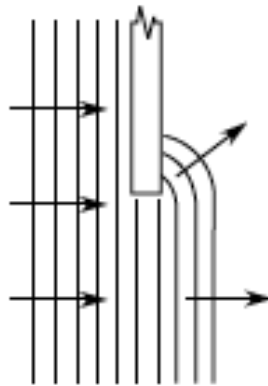
$$n = \frac{c_{\text{vacuum}}}{c_{\text{medium}}}$$

$$c_{\text{medium}} = \frac{c_{\text{vacuum}}}{n}$$

$$c_{\text{water}} = \frac{3 \cdot 10^8 \frac{\text{m}}{\text{s}}}{1.3300} = 2.26 \cdot 10^8 \text{ m/s}$$

### Diffraction

*Diffacted light* is light whose path has been changed by passing around corners or through narrow slits. As a consequence of *Huygens' principle*, each edge of a single diffraction slit (shown in Fig. 20) acts as a source of light waves and is capable of producing interference. For diffraction of monochromatic light by a single slit, the  $m$ th order reinforcements and cancellations are the same as predicted by Eq. 40 through Eq. 43.



**Figure 20: Diffraction Around a Corner**

The same equations can also be used for a *diffraction grating*, a transparent sheet containing numerous slits (scratches) spaced a distance  $d$  (known as the *grating space* or *grating constant*) apart and typically found in a *diffraction grating spectrometer*.<sup>21,22</sup> The angle at which the

<sup>21</sup> The number of slits per centimeter ranges from 400 to 6000.

<sup>22</sup> While the number of slits (one, two, or hundreds) does not affect the position of the image, it does affect the brightness of the image.

### *Illumination*

spectrometer must be turned to view an  $m$ th order image can be calculated from  $\sin \theta = y/s \approx y/x$ . (The term  $s$  is the slit distance, normally  $d$ .) The maximum number of orders of interference produced can be determined by setting  $\theta$  equal to  $90^\circ$ .

#### **Equation 40: In Phase Image**

$$\sin \theta = \frac{m\lambda}{d} \quad [\text{in phase}]$$

#### **Equation 41: Out of Phase Image**

$$\sin \theta = \frac{(2m+1)\lambda}{2d} \quad [\text{out of phase}]$$

### **Interference**

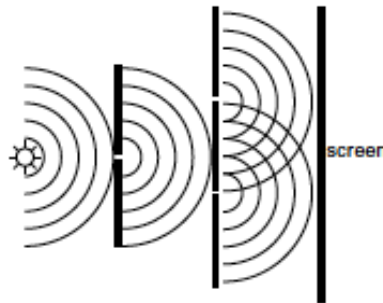
*Interference* (also known as *cancellation*) occurs when two waves combine so that one subtracts from the other. It is also possible for two waves to add to each other, which is known as *reinforcement* or *superposition*.

Most light sources emit light waves with random phase relationships. However, a source of parallel rays can be obtained by allowing light to pass through a narrow slit. According to *Huygens' principle* (also known as *Huygens' construction*), each point in a wave front can be considered as a source of waves. Therefore, the slit will act as a light source, as shown in Fig. 21. The circular lines represent the wave maxima (crests) and the spaces represent the wave minima (troughs).

Interference can be obtained by allowing the parallel rays to pass through two additional slits, which themselves act as two sources. However, these secondary sources are in phase because the light was derived from a single primary slit.



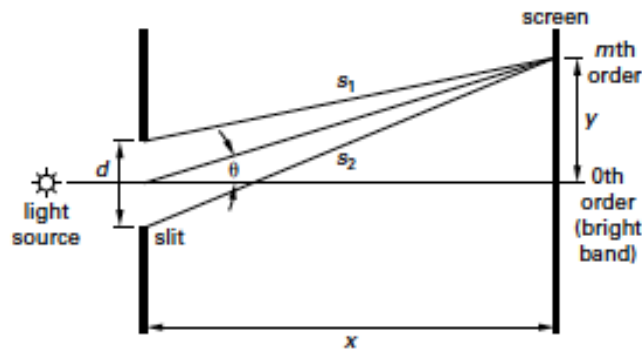
### Illumination



**Figure 21: Huygens' Principle**

### Interference from Slits

The visual effect on a distant screen of combining two in phase sources will be regions of darkness and light. (The arrangement shown in Fig. 22 is known as *Young's experiment*.) A bright spot or band means that two wave crests or two wave troughs coincide (i.e., are in phase at that point) and thereby reinforce each other. If a trough coincides with a crest, the two sources are 180 degrees out of phase, and the result is a dark spot or band.



**Figure 22: Interference**

For reinforcement (i.e., a bright band) to occur, the difference in path lengths must be a whole number,  $m$ , of wavelengths. The number  $m$  is known as the *order of interference*. The central bright band is of the zeroth order, and there are two of each  $m$ th order image, one on each side of the center. When  $x \gg d$ , small angle approximations are valid and  $y/x = (s_2 - s_1)/d$ . For the  $m$ th reinforcement (light band, maximum, etc.),

*Illumination***Equation 42: The  $m$ th Order In Phase**

$$s_1 - s_2 = m\lambda \quad [\text{in phase}]$$

**Equation 43: Distance from Zeroth Order**

$$y = \frac{m\lambda x}{d} \quad [\text{in phase}]$$

Between the bright bands are dark bands of cancellation. For cancellation, the difference in path lengths is an odd number of half wavelengths. For the  $m$ th cancellation (dark band, minimum, etc.),

**Equation 44: The  $m$ th Order Out of Phase**

$$s_1 - s_2 = \frac{(2m + 1)\lambda}{2} \quad [\text{out of phase}]$$

**Equation 45: Distance from Zeroth Order**

$$y = \frac{(2m + 1)\lambda x}{2d} \quad [\text{out of phase}]$$

**Example 6**

A screen is placed 2.4 m from a sodium gas discharge tube that is emitting two beams of in-phase light ( $\lambda = 5.893 \times 10^{-7}$  m) separated by 0.0005 m. What is the distance from the central image (zeroth order) to the first reinforcement?

*Solution*

From Eq. 43 with  $m = 1$ ,

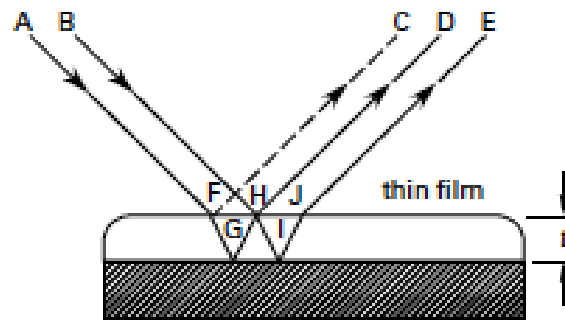
### Illumination

$$\begin{aligned}
 y &= \frac{m \lambda}{d} \\
 &= \frac{(1)(5.893 \times 10^{-7} \text{ m})(2 \text{ m})}{0.0005 \text{ m}} \\
 &= 0.0028 \text{ m}
 \end{aligned}$$

### Interference from Thin Films

Interference occurs when light passes through thin films (walls of soap bubbles, layers of oil on water, etc.).<sup>23</sup> If the incident beam is white light (i.e., is composed of light of all colors), each wavelength will produce its own interference pattern, resulting in a rainbow effect.

Figure 23 shows a solid reflective surface covered by a transparent film of thickness  $t$ . Ray D is composed of a partial reflection of ray B and the remainder of ray A. Because of the 180 degree phase reversal at point G, cancellation along ray D requires that the path FGH be an integral number of wavelengths. However, the path length is also approximately equal to twice the film thickness.



**Figure 23: Thin Film Interference**

Equation 47 and Eq. 48 define the relationships for the  $m$ th order reinforcement and cancellation, respectively. It is essential that the wavelength in the film,  $\lambda_{\text{film}}$ , be used, not the free space wavelength (see Eq. 48).

<sup>23</sup> A circular pattern of dark bands known as Newton's rings is created by interference in a film of thin air trapped between a flat reflecting surface and a plano-convex lens placed flat side up.

## *Illumination*

### **Equation 46: Thin Film Interference In Phase**

$$\left(m + \frac{1}{2}\right) \lambda_{\text{film}} = 2t \quad [\text{in phase}]$$

### **Equation 47: Thin Film Interference Out of Phase**

$$m \lambda_{\text{film}} = 2t \quad [\text{out of phase}]$$

### **Equation 48: Wavelength Inside a Film**

$$\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}}$$

## **Lighting Design Methods**

Various design methods exist for determining the illuminance in a general area or at particular point. These methods are dependent on the source of light, the type of lighting used, and the tasks to be accomplished in the space. Both direct and indirect (from reflection) light sources are considered in the methods.<sup>24</sup>

General area calculations focus on the average illuminance, considering all sources—daylight, reflections, interior lighting sources (luminaries). When daylight is part of the calculation, the lumen method is appropriate. If only interior lighting sources are involved, the zonal cavity method is appropriate. The lumen and zonal cavity methods differ in the use of factors. However, the terms “lumen” and “zonal cavity” are often used interchangeably.

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<sup>24</sup> Details of these calculations are extensive. If interested, Chap. 8 and Chap. 9 of the IESNA Lighting Handbook (Ninth Edition, 2000, Illuminating Engineering Society of North America, New York), Ref [D], provide the details.

## *Illumination*

If lighting for specific task accomplishment is the design goal, the point method is used. This method calculates the illuminance at a point or surface using variations of the laws as described earlier. Supplementary sources, controlled separately from the main lighting source, are often incorporated into designs requiring such task lighting.

### **Daylight**

Daylight has unique spectra and light-level distributions that distinguish it from artificial lighting. Daylight varies daily and seasonally due to *movement of the sun*.<sup>25</sup> Variations in daylight also occur from weather and local environmental factors. As a result, the CIE has developed three spectral radiant power distributions for daylight. They are known by the symbols  $D_{55}$ ,  $D_{65}$ , and  $D_{75}$ , which stand for daylight at 5500 K, 6500 K, and 7500 K respectively (see color temperature from earlier).

Using the standard power distributions of daylight, mean illuminance from the sun can be determined by factoring in the site location, the time of desired illumination, the solar position, skylight and groundlight contributions, and human response factors. Such calculations can be difficult and complicated.

### **Daylight Calculation Methods**

The daylight level of illuminance can be calculated using either a computer method or a manual method. Computer methods all involve a software-based system in which the operator provides the necessary input and design restrictions. Software-based systems generally utilize the CIE standard distributions and can be customized with any number of input factors to determine the mean illuminance. Computer methods are the most accurate, and depending upon the level of detail and accuracy, can be expensive. [J]

A manual method is appropriate for first-order calculations on direct and reflected components of light. Some examples of manual methods are the lumen method, used for both toplighting and sidelighting, and the daylight factor method.<sup>26</sup>

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<sup>25</sup> The phrase “movement of the sun” is used here to reflect that many equations for lighting assume a static Earth with the sun movement as the entering argument for the equations.

<sup>26</sup> The results from such calculations may be required to augment the electric lighting calculations, normally the province of the electrical engineer.

## Illumination

### Daylight Lumen Method

The daylight lumen method<sup>27</sup> uses the following steps:

*step 1:* Calculate exterior illuminances using the CIE standard spectral radian power distributions modified for the factors mentioned in Daylight Calculation Methods.

*step 2:* Accounting for fenestration, determine the net transmittance into the room. The fenestration is defined as any opening or arrangement of openings exposing daylight. Fenestration determines the level of light available after accounting for furniture placement, reflectivity of walls and components (i.e., light loss factors), and additional factors depending upon the level of complexity desired.

*step 3:* *Coefficients of utilization* (CU) are applied. CUs are ratios of the interior to exterior horizontal illuminances. For the toplighting lumen method, the coefficients provide the illuminance on the workplane. For the sidelighting lumen method, coefficients are given for five predetermined points.

*step 4:* Calculate the product of the factors from steps 1, 2, and 3 to determine the interior illuminance.

These steps are expressed in the general equation for the lumen method, given as Eq. 49.

#### Equation 49: Average Incident Illuminance

$$E_i = E_{xh} t_{\text{net}} (CU) \left( \frac{A_s}{A_w} \right)$$

The average incident illuminance, given in lux, on the workplane is represented by  $E_i$ . The exterior horizontal illuminance available on the skylights is given as  $E_{xh}$ . The symbol  $\tau_{\text{net}}$  represents the net transmittance through the skylights and light well, including loss factors for any control devices that limit the light and maintenance (i.e., losses due to dirt or other environmental factors that are limiting the light transmission). The area of the skylights and the area of the workplane are

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<sup>27</sup> The lumen method is similar to the zonal method used for electric lighting. Given the similarities, the details and equations are explained in the following section on lighting calculations.

## Illumination

represented by  $A_s$  and  $A_w$ , respectively. Most of the terms in Eq. 49 are either given in the design or can be determined from tabulated values based on design criteria. For example,  $E_{xh}$  can be calculated using one of the daylight calculation methods, or given for the design. The net transmittance is calculated by Eq. 50.

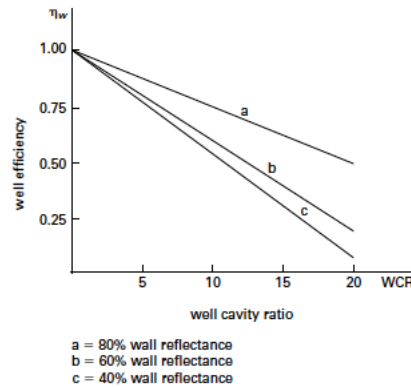
### Equation 50: Net Transmittance

$$t_{d \text{ or } D} = (T_{d \text{ or } D}) h_w R_a T_c (\text{LLF})$$

The net transmittance,  $\tau$ , can be either diffuse ( $d$ ) or direct ( $D$ ) depending upon the transmittance term used,  $T_d$  or  $T_D$ . This data is either provided by the manufacturer or it is measured. The ratio of the net skylight area to the gross skylight area,  $R_a$ , is calculated from the design data. Figure 24 shows an example of a graph used to find the well efficiency,  $\eta_w$ . The wall reflectance is generally tabulated or given in manufacturing data. The well cavity ratio (WCR) is calculated by Eq. 51. The wall reflectance and the WCR comprise the data necessary to calculate well efficiency, as seen in Fig. 24.

### Equation 51: Well Cavity Ratio

$$\text{WCR} = \frac{5H(L + W)}{LW}$$



**Figure 24: Well Efficiency**

## Illumination

The light loss factor, LLF, is typically given in a table. An example is shown in Table 8.

**Table 8: Example Light Loss Factors**

location	glazing position		
	vertical	sloped	horizontal
clean	0.9	0.8	0.7
industrial	0.8	0.7	0.6
extremely dirty	0.7	0.6	0.5

CU is a tabulated factor and is based on the design conditions. For the toplighting method, ceiling and wall reflectance are two of the input criteria, and are provided by manufacturing (or measured) data. Another common input criteria is the room cavity ratio (RCR). The room cavity ratio is calculated using Eq. 52.

**Equation 52: Room Cavity Ratio**

$$RCR = \frac{5H_{c-wp} (L + W)}{LW}$$

Both Eq. 51 and Eq. 52 provide a unique ratio based on the height. This ratio is used to estimate the CU value. The larger the RCR value, the less light available in any given situation. Table 9 shows examples of CU values. All such tables are based on the space to mounting height ratio,<sup>28</sup> a lambertian distribution<sup>29</sup> from the skylight, and an assumed floor reflectance. Care must be taken to select the correct table for the given design constraints.

**Table 9: Coefficient of Utilization for Sky Lighting**

ceiling reflectance (%)	RCR	wall reflectance 50%	wall reflectance 10%
80	0	1.19	1.19
	5	0.67	0.53
	10	0.52	0.40
20	0	1.04	1.04
	5	0.61	0.51
	10	0.49	0.40

<sup>28</sup> This refers to the mounting height of the skylight opening from the workplane.

<sup>29</sup> A lambertian distribution is one following Lambert's cosine law.



## *Illumination*

The general lumen method represented by Eq. 49 can now be made specific for toplighting in overcast skies by Eq. 53, and for clear skies by Eq. 54.

### **Equation 53: Overcast Skylight Illuminance**

$$E_i = E_{xh,sky} t_d (CU) \left( \frac{A_s}{A_w} \right)$$

### **Equation 54: Clear Skies Skylight Illuminance**

$$E_i = \left( E_{xh,sky} t_d + E_{xh,sun} t_d \right) (CU) \left( \frac{A_s}{A_w} \right)$$

For the lumen method for sidelighting, Eq. 49 is modified to eliminate the area ratio and to replace the exterior horizontal illuminance term,  $E_{xh}$ , with the term for exterior vertical illuminance on the window wall,  $E_{xv}$ . The resulting equation is shown as Eq. 55. The individual terms of the equation are calculated the same way as for the lumen method for toplighting. The assumptions embedded in the sidelighting method are: the ceiling cavity has a 70% reflectance, the room cavity has a 50% reflectance, and the floor cavity has a 30% reflectance.<sup>30</sup>

### **Equation 55: Lumen Method Sidelighting**

$$E_i = E_{xv} t_{net} (CU)$$

## **Daylight Factor Method**

The daylight factor method is a low precision process. It is used with known illuminance distributions, such as the standard three from the CIE. The data for calculating the illuminance is given in a table. The daylight factor, DF, is a ratio of the illuminance at a point to the illuminance from the unobstructed sky. It is shown in Eq. 56. This unitless factor is then multiplied by the standard illuminance distribution to obtain the estimated illuminance in the space.

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<sup>30</sup> The ceiling cavity is from the top of the windows to the ceiling. The floor cavity is from the windowsill to the floor. The room cavity is the space in between the two.

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### **Equation 56: Daylight Factor Method**

$$DF = SC + ERC + IRC$$

The term SC represents the sky component, ERC is the externally reflected component, and IRC is the internally reflected component.

### **Electric Lighting Principles**

Lighting calculations are approximations of complex physical processes and are designated by the intended application of the calculation. Specific applications include daylighting, luminaire design, runway design, roadway design, and security lighting. Basic equations govern the first order calculations and are a good basis for initial design. Those methods previously discussed for daylight are applicable with slight adjustments to the equations. The two most important lighting methods are the lumen method and the zonal cavity method. Both methods assume that air is non-absorbing and non-scattering. The basic equation, applicable to both methods, is given by Eq. 57.

### **Equation 57: Lumen & Zonal Method Equation**

$$E_{avg} = \frac{\text{total flux onto workplace}}{\text{workplace area}}$$

### **Lumen Method**

The lumen method accounts for the total lumens projected by a luminaire to the workplane derated with a coefficient of utilization factor. The CU factor accounts for the efficiency of the luminaire, the distribution of the light from the luminaire (which is highly dependent upon the shape and properties of the enclosing material), and the reflective properties of the surrounding space. The average initial expected illuminance is given by Eq. 58.

### **Equation 58: Average Expected Illumination**

$$E_{initial} = \frac{L_{total}(CU)}{A_w}$$

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The luminance emitted is represented by  $L$  with units of lumens and is determined from manufacturing data. The CU factor must be given (by the manufacturer for a given luminaire and assumed use), estimated (using reflectance calculations for the given material in a space),<sup>31</sup> or calculated using the zonal cavity method.  $A_w$  is the area that the designer is attempting to light to a given level. If the area is given in square meters, then the average illuminance units are lux (lumen per  $m^2$ ). If the area is given in square feet, then the average illuminance units are footcandles (lumen per  $ft^2$ ).

Lighting dims over time due to a variety of factors, including light loss due to dirt accumulation. To account for this loss, a light loss factor (LLF) is introduced. This is another derating factor whose values are normally tabulated. The LLF consists of recoverable losses (such as dirt accumulations) and nonrecoverable losses (such as luminaire physical degradation).<sup>32</sup> The designer's goal is to maintain an average minimum lighting level over time. To do this, the LLF must be included as in Eq. 59.

### **Equation 59: Illumination Maintained (with Losses)**

$$E_{\text{maintained}} = \frac{L_{\text{total}}(CU)(LLF)}{A_w}$$

The total lumens are usually provided by a number of luminaires. To calculate the number of lighting devices needed, assuming each provides the same lumens, use Eq. 60 and solve for  $N_{\text{lights}}$ .

### **Equation 60: Illumination Maintained per Lights**

$$E_{\text{maintained}} = \frac{N_{\text{lights}} L_{\text{per light}} (CU)(LLF)}{A_w}$$

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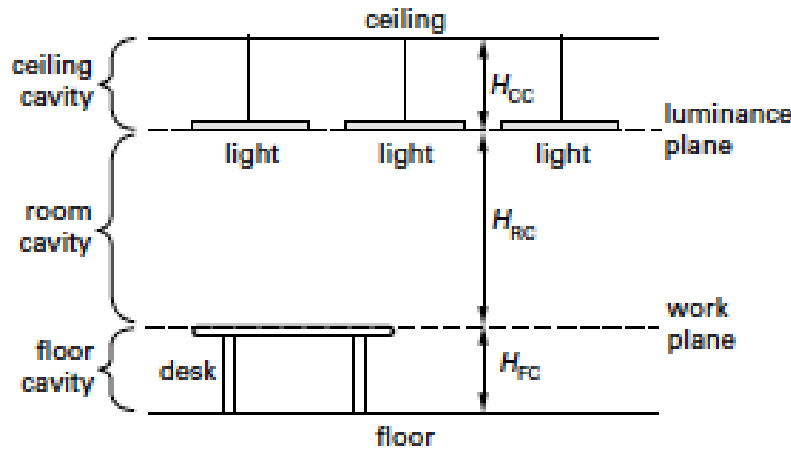
<sup>31</sup> The calculation of effective cavity reflectances is not covered in this text. The value is usually given, and otherwise the equation for calculating the reflectance is provided. The use of such reflectances can enhance a lighting calculation but should be left to computer-based methods. Effective cavity reflectances are also called base reflectances for entering arguments into a zonal cavity table to calculate a coefficient of utilization.

<sup>32</sup> Nonrecoverable losses include degradation to the luminaire from ambient temperature effect, supply voltage level, ballast factor, and luminaire surface depreciation. Recoverable losses include degradation from room surface dirt depreciation (RSDD), lumen lamp depreciation (LLD), lamp burnout factor (LBO), and luminaire dirt depreciation (LDD).

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### Cavity Ratios (Zonal Method)

The amount of light exchanged between the top of a space and an area below or above it is a function of the proportions of its length, width, and height. The cavity ratio provides a single number approximating the effects. The concept is illustrated in Fig. 24. The zonal cavity method uses the cavity ratio to calculate coefficients of utilization.



**Figure 25: Zonal Cavity Terminology**

A room or space is divided into three cavities. The cavity ratio (CR) is calculated using

#### Equation 61: Cavity Ratio

$$CR = \frac{5H_{xx}(L + W)}{LW}$$

When  $H_{xx}$  is replaced with  $H_{CC}$ , Eq. 61 gives the ceiling cavity ratio (CCR). When  $H_{xx}$  is replaced with  $H_{FC}$ , Eq. 61 gives the floor cavity ratio (FCR). When  $H_{xx}$  is replaced with  $H_{RC}$ , Eq. 61 gives the room cavity ratio (RCR).

If a room is an irregular shape, the perimeter of the room and the area of its base are used, along with an adjusted factor, which is  $-2.5$  because the perimeter of the rectangular room in Eq. 61 is actually  $2(L + W)$ , as shown in Eq. 62.

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**Equation 62: Cavity Ratio—Perimeter**

$$CR = \frac{2.5H_{xx}P}{A_{base}}$$

Using the reflectance of the appropriate surface (the ceiling, the floor, or the floor of the cavity itself—for example, the workplane of the room cavity) as the base reflectance, and knowing or estimating the wall reflectance, and finally calculating the appropriate cavity ratio, one has the input arguments necessary to use a zonal cavity table to determine the coefficient of utilization. See Table 10 for an example.<sup>33</sup> Select the base reflectance (either floor or ceiling), then select the wall reflectance. Move down the column, stopping at the row associated with the appropriate cavity ratio. The CU appears, as a percentage, in the intersection of the column and the row. For example, if the ceiling reflectance is 80% and the wall reflects at 50%, the CU is 71% for a cavity ratio of 0.6.

**Table 10: Coefficient of Utilization (%)**

base reflectance (%)	90	90	80	80
wall reflectance (%)	50	0	50	0
cavity ratio				
0.2	86	82	77	72
0.6	80	73	71	63
1.0	75	62	67	55

<sup>33</sup> Such tables also assume a length-to-width ratio for the luminaire. Care should be taken to ensure the correct table is being used for the design conditions.

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### NOTE

Electrical refers to something related to electricity while “electric” refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

Numerous articles throughout the NEC contain requirements for lighting, lighting units, lighting systems, and luminaires. [Additionally, recommend using the index for [C], [D], and [E] finding a listing of applicable standards.]

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# Illumination

## Appendix A: Units, Symbols, and Defining Equations

symbol	definition	units	unit symbols
$\alpha$	absorptance $\alpha = \Phi_a / \Phi_i$	—	—
$E$ $E_e$ (radiometric)	irradiance $E = d\Phi / dA$	(SI) watt per square meter (cgs) watt per square centimeter	$\text{W}/\text{m}^2$ $\text{W}/\text{cm}^2$
$E$ $E_v$ (photometric)	lluminance (“illumination” is deprecated) photometric illuminance $E = d\Phi / dA$	(SI) lux (cgs) phot (U.S.) footcandle	$\text{lx} = \text{lm}/\text{m}^2$ $\text{ph} = \text{lm}/\text{cm}^2$ $\text{fc} = \text{lm}/\text{ft}^2$
$\epsilon$	emissivity, spectral-total hemispherical $\epsilon = M / M_{\text{blackbody}}$	— —	— —
$\epsilon(\theta, \phi, T)$	emissivity, spectral-total directional $\epsilon(\theta, \phi, T) = L(T) / L_{\text{blackbody}}(T)$	—	—
$\epsilon(\theta, \phi, \lambda, T)$	emissivity, spectral directional $\epsilon(\theta, \phi, \lambda, T) = L(\theta, \phi, \lambda, T) / L_{\lambda, \text{blackbody}}(\lambda, T)$	—	—
$\epsilon(\lambda, T)$	emissivity, spectral hemispherical $\epsilon(\lambda, T) = M_\lambda(\lambda, T) / M_{\lambda, \text{blackbody}}(\lambda, T)$	—	—
$I$ $I_e$ (radiometric)	radiant intensity $I = d\Phi / d\omega$	(SI) watt per steradian	$\text{W}/\text{sr}$
$I$ $I_v$ (photometric)	luminous intensity (candle power) $I = d\Phi / d\omega$	candela (lumen per steradian)	cd
$K$	luminous efficacy $K = \Phi_v / \Phi_e$	lumen per watt	$\text{lm}/\text{W}$
$L$ $L_e$ (radiometric)	radiance $L = d^2\Phi / d\omega dA \cos\theta = dI / dA \cos\theta$	(SI) watt per steradian per square meter watt per steradian per square centimeter	$\text{W}/\text{sr}\cdot\text{m}^2$ $\text{W}/\text{sr}\cdot\text{cm}^2$
$L$ $L_v$ (photometric)	luminance $L = d^2\Phi / d\omega dA \cos\theta = dI / dA \cos\theta$	candela per unit area stib (SI) nit footlambert (deprecated) lambert (deprecated) apostib (deprecated)	e.g., $\text{cd}/\text{in}^2$ $\text{sb} = \text{cd}/\text{cm}^2$ $\text{nt} = \text{cd}/\text{m}^2$ $\text{fl} = \text{cd}/\pi\cdot\text{ft}^2$ $\text{L} = \text{cd}/\pi\cdot\text{cm}^2$ $\text{asb} = \text{cd}/\pi\cdot\text{m}^2$
$\lambda$	wavelength used as subscript or argument of a spectral function, to specify a wavelength, as in $Q_\lambda$ for spectral concentration or $K(\lambda)$ for a function of wavelength, especially when quantities are restricted to narrow wavelength bands		
$M$ $M_e$ (radiometric)	radiant exitance radiant flux density at a surface radiant emittance (deprecated) $M = d\Phi / dA$	(SI) watt per square meter (cgs) watt per square centimeter	$\text{W}/\text{m}^2$ $\text{W}/\text{cm}^2$
$M$ $M_v$ (photometric)	luminous exitance luminous flux density at a surface luminous emittance (deprecated) $M = d\Phi / dA$	lumen per square meter lumen per square foot	$\text{lm}/\text{m}^2$ $\text{lm}/\text{ft}^2$

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symbol	definition	units	unit symbols
$\Phi_a$	absorbed flux	—	—
$\Phi_i$	incident flux	—	—
$\Phi_r$	reflected flux	—	—
$\Phi_t$	transmitted flux	—	—
$\Phi$	radiant flux	(SI) watt	W
$\Phi_e$ (radiometric)	$\Phi = dQ/dt$	(cgs) erg per second	erg/s
$\Phi$	luminous flux	(SI) lumen	lm
$\Phi_v$ (photometric)	$\Phi = d\Phi/dt$		
$Q$	radiant energy	(SI) joule	J
$Q_e$ (radiometric)		(cgs) erg	erg
		calorie	cal
		kilowatt-hour	kW·h
$Q$	luminous energy (quantity of light)	(SI) lumen-second (talbot)	lm·s
$Q_v$ (photometric)	$\int_{360}^{800} K(\lambda)Q_e\lambda d\lambda$	lumen-hour	lm·h
$\tau$	transmittance $\tau = \Phi_t/\Phi_i$	—	—
$w$	radiant energy density	(SI) joule per cubic meter	J/m <sup>3</sup>
$w_e$ (radiometric)	$w = dQ/dV$	(cgs) erg per cubic centimeter	erg/cm <sup>3</sup>
$V$	luminous efficiency $V = K/K_{\max}$	—	—



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## Appendix B: Luminous Conversion Factors

multiply	by	to obtain
apostilb (international)	10	mL (deprecated)
apostilb (German Hefner)	11.11	mL (deprecated)
blondel	10	mL (deprecated)
cd/m <sup>2</sup> or nt	0.000645	cd/in <sup>2</sup>
cd/m <sup>2</sup> or nt	0.0929	cd/ft <sup>2</sup>
cd/m <sup>2</sup> or nt	0.001	stilb or cd/cm <sup>2</sup>
cd/m <sup>2</sup> or nt	0.2919	fL (deprecated)
cd/m <sup>2</sup> or nt	0.3142	mL (deprecated)
cd/in <sup>2</sup>	1550	cd/m <sup>2</sup> or nt
cd/in <sup>2</sup>	144	cd/ft <sup>2</sup>
cd/in <sup>2</sup>	0.155	stilb or cd/cm <sup>2</sup>
cd/in <sup>2</sup>	452	fL (deprecated)
cd/in <sup>2</sup>	487	mL (deprecated)
cd/ft <sup>2</sup>	10.76	cd/m <sup>2</sup> or nt
cd/ft <sup>2</sup>	0.00694	cd/in <sup>2</sup>
cd/ft <sup>2</sup>	0.00108	stilb or cd/cm <sup>2</sup>
cd/ft <sup>2</sup>	3.142	fL (deprecated)
cd/ft <sup>2</sup>	3.382	mL (deprecated)
lambert (L)	0.001	mL (deprecated)
stilb or cd/cm <sup>2</sup>	10,000	candela/m <sup>2</sup> or nt
stilb or cd/cm <sup>2</sup>	6.45	cd/in <sup>2</sup>
stilb or cd/cm <sup>2</sup>	929	cd/ft <sup>2</sup>
stilb or cd/cm <sup>2</sup>	2919	fL (deprecated)
stilb or cd/cm <sup>2</sup>	3142	mL (deprecated)
fL (deprecated)	3.426	cd/m <sup>2</sup> or nt
fL (deprecated)	0.00221	cd/in <sup>2</sup>
fL (deprecated)	0.3183	cd/ft <sup>2</sup>
fL (deprecated)	0.00034	stilb or cd/cm <sup>2</sup>
fL (deprecated)	1.076	mL (deprecated)
mL (deprecated)	3.183	cd/m <sup>2</sup> or nt
mL (deprecated)	0.00205	cd/in <sup>2</sup>
mL (deprecated)	0.2957	cd/ft <sup>2</sup>
mL (deprecated)	0.00032	stilb or cd/cm <sup>2</sup>
mL (deprecated)	0.929	fL (deprecated)
mL (deprecated)	1000	L

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multiply	by	to obtain
erg	$10^7$	watt-second
footcandles	10.76	lx or $\text{lm}/\text{m}^2$ or metercandle
footcandles	0.00108	phot or $\text{lm}/\text{cm}^2$
footcandles	1.076	milliphot
lumen	1/683	light-watt
lumen-hour	60	lumen-minutes
light-watt	683	lumen
lux	0.0929	footcandles
lux	0.0001	phot or $\text{lm}/\text{cm}^2$
lux	0.1	milliphot
phot	929	footcandle or $\text{lm}/\text{ft}^2$
phot	10,000	lx or $\text{lm}/\text{m}^2$ or metercandle
phot	1000	milliphot
milliphot	0.929	footcandle or $\text{lm}/\text{ft}^2$
milliphot	10	lx or $\text{lm}/\text{m}^2$ or metercandle
milliphot	0.001	phot or $\text{lm}/\text{cm}^2$
watt-second	$10^{-7}$	erg