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Electrical Power

Part II: Distribution Systems

by

John A Camara, BS, MS, PE, TF

Course 531

4 PDH (4 Hours)

PO Box 449
Pewaukee, WI 53072
(888) 564 - 9098
eng-support@edcet.com

Electrical Power: Part II

Nomenclature[1]

A	ABCD parameter	-
a	phase	-
A	area	m^2
B	ABCD parameter	-
B	magnetic flux density	T
B	magnetic flux density	T
B	susceptance	S, Ω^{-1}, or mho
b	phase	-
c	speed of light	m/s
C	capacitance	F
C	ABCD parameter	-
c	phase	-
D	ABCD parameter	-
D	distance	m
E	electric field strength	V/m
E	energy	J
E	voltage (generated)	V
f	frequency	Hz, s^{-1}, cycles/s
f_{droop}	frequency droop	Hz/kW
G	conductance	S, Ω^{-1}, or mho
GMD	geometric mean distance	m
GMR	geometric mean radius	m
h	specific enthalpy	kJ/kg
I	effective or DC current	A
I	rms phasor current	A
K	correction factor	-
K	skin effect ratio	-
l	length	m
L	inductance	H
m	mass	kg

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related, and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in all "Parts" of this complete course. For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

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M	mutual inductance	H
n	Steinmetz exponent	-
N	number of turns	-
n_s	synchronous speed	r/min or min^{-1}
p	pressure	Pa
P	number of poles	-
P	power	W
pf	power factor	-
pu	per unit	-
Q	heat	J
r	radius	m
R	resistance	Ω
s	specific entropy	kJ/kg·K
S	apparent power	kVA
SWR	standing wave ratio	-
T	temperature	ºC or K
v	wind velocity	km/hr
V	effective or DC voltage	V
v	velocity (speed)	m/s
V	rms phasor voltage	V
V_{droop}	voltage droop	V/kVAR
VR	voltage regulation	-
W	work	kJ
X	reactance	Ω
x	variable	-
Y	admittance	S, Ω^{-1}, or mho
y	admittance per unit length	S/m, 1/Ω·m Ω^{-1}, or mho/m [℧ / m]
Z	impedance	Ω
z	impedance per unit length	Ω/m
Z_0	characteristic impedance	Ω

Electrical Power: Part II

Symbols

α	turns ratio	-
α	attenuation constant	Np/m
α	thermal coefficient of resistance	1/°C
β	phase constant	rad/m
γ	propagation constant	rad/m
Γ	reflection coefficient	-
δ	skin depth	m
Δ	change, final minus initial	-
ε	permittivity	F/m
ε_0	free-space permittivity	8.854×10^{-12} F/m
ε_r	relative permittivity	-
η	efficiency	-
θ	phase angle	rad
κ	coupling coefficient	-
μ	permeability	H/m
μ_o	free-space permeability	1.2566×10^{-6} H/m
μ_r	relative permeability	-
ξ	ratio of radii	-
ρ	resistivity	$\Omega\cdot$m
σ	conductivity	S/m
ω	armature angular speed	rad/s

Subscripts

ϕ	phase
0	zero sequence
0	characteristic
0	free space (vacuum)
0,o	initial (zero value)
1	positive sequence
1	primary
2	negative sequence
2	secondary
ab	a to b
AC	alternating current
avg	average
bc	b to c
c	controls or critical
c	core
C	capacitor
ca	c to a
Cu	copper
d	direct
DC	direct current
e	eddy current
e	equivalent
eff	effective
ext	external
f	final / frequency
fl	full load
g	generator
h	hysteresis
int	internal
l	line
l	line
l	per unit length

L	inductor
ll	line-to-line
m	motor
m	maximum
m	mutual
max	maximum
n	neutral
nl	no load
O	origin
oc	open circuit
p	phase
p	primary
ps	primary to secondary
pu	per unit
q	quadrature
R	receiving end
R	resistance
s	synchronous
s	secondary
S	sending end
sc	short circuit
sys	system
t	terminal
w	wave

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Electrical Power: Part II

INTRODUCTION

Although this is a five part course, each individual part is meant to be stand-alone should one be interested in that topic. The overall purpose of the course is to provide an overview of electric power from generation, through the various distribution systems, including the vital transformer links that change the voltage from the high voltage required for minimum losses during transmission to medium- and low-voltage for the end-users. Additionally, the transmission lines connecting the system are covered. And, finally, the rule from the National Electric Safety Code® (NESC®) that govern it all completes the overview.

Part I, Generation, the more common type of plants producing the power. The basics of alternating current and direct current generators is explained include the principles of parallel operation. Finally, energy management and power quality are covered.

Part II, Distribution Systems, covers the classification of such systems, how the common neutral is utilized, overhead and underground distribution, along with fault analysis methods.

Part III, Transformers, informs on power transformers, their ratings, voltage regulation, testing methods and parameters used to analyze both transformers and transmission lines.

Part IV, Transmission Lines, discusses the electrical parameters of such line: resistance, inductance, and capacitance. Important effects such as the skin effect and reflection are explained. This part completes with an explanation of models for each type of transmission line: short, medium, and long.

Part V, The National Electrical Safety Code, covers organization of the code and some of the multitude of requirements for the transmission of electrical power.

The information is primarily from the author's books, Refs. [A] and [B] with the NESC information from the Handbook covering the code, Ref. [C]. The coverage of the NESC does not include end-users buildings—this is covered by the NEC, Ref. [D]. Information useful in many aspects of electric engineering may be found in [E] and [F] as well as the appendices.

Electrical Power: Part II

FUNDAMENTALS

The electric distribution system is the collection of circuitry, high-voltage switchgear, transformers, and related equipment that receives high voltage from the source and delivers it at lower voltages. Its function is to receive power from large bulk sources, that is, generation sources, and distribute it to users at the voltage levels and degrees of reliability required. A hypothetical distribution system one-line diagram is shown in Fig. 1. The electric utility may consider the distribution system as that portion of Fig. 1 from the distribution substation to the consumer. The symbols used are explained in Fig. 2.

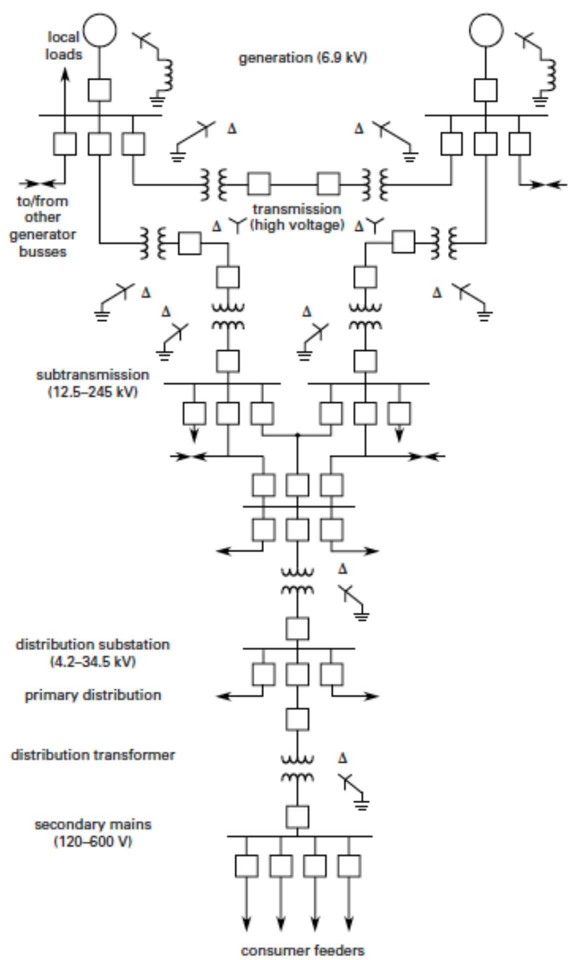

Figure 1: Distribution System

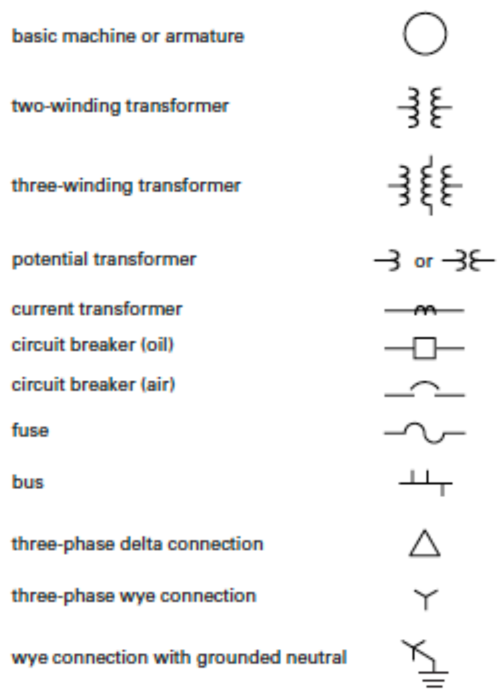

Figure 2: Common Distribution Symbols[2]

One-line diagrams can be used for balanced three-phase systems because the per-phase values are equal. If a neutral line exists, it is not shown. If the diagram is used for load studies, the circuit breaker, fuse, and other switching device locations are not shown because they are of no concern. If the diagram is used for stability studies or fault analysis, the locations and characteristics of circuit breakers, fuses, relays, and other protective devices are shown.

At the generation level in Fig. 1, the voltage is approximately 6.9 kV. The synchronous generators are shown with their neutrals connected through impedances designed to limit surges in case of a fault in the generator circuit. One or more generators may be attached to a given power bus.

At the *transmission level*, the transmission lines connect the various generators to one another and to the sub-transmission lines. At the *sub-transmission level*, the voltage range is 12.5 kV to 245 kV. The most commonly used voltages in order of usage are 115 kV, 69 kV, 138 kV, and 230 kV. The sub-transmission lines are usually in grid form. The grid form allows connections between

[2] Symbols of all sorts are standardized in Ref. [E]. The many electrical letter symbols are standardized in Ref. [F].

input busses and various paths to each distribution substation, thus increasing reliability. A minimum of two switchable inputs to each substation input bus is normally used.

At the *primary distribution level*, the voltage range is 4.2 kV to 34.5 kV. The most commonly used voltages are 12.5 kV, 25 kV, and 34.5 kV. The actual voltage level is controlled with taps on the substation transformers. The distribution substation supplies several distribution transformers connected in a radial, tree, loop, or grid system. The distribution transformers are mounted on poles, grade-level pads, or underground near the user (a substation may be dedicated to a single large user). Secondary mains operate at a voltage range of 120 V to 600 V.[3]

CLASSIFICATION OF DISTRIBUTION SYSTEMS

Distribution systems are classified in a number of ways, including

- current (AC or DC)
- voltage (120 V, 12.5 kV, 15 kV, 34.5 kV, etc.)
- type of load (residential, commercial, lighting, power, etc.)
- number of conductors (two-wire, three-wire, etc.)
- construction type (overhead or underground)
- connection type

Connection types include radial (seldom used), loop (see Fig. 3), network, multiple (see Fig. 4), and series (used primarily for street lighting).

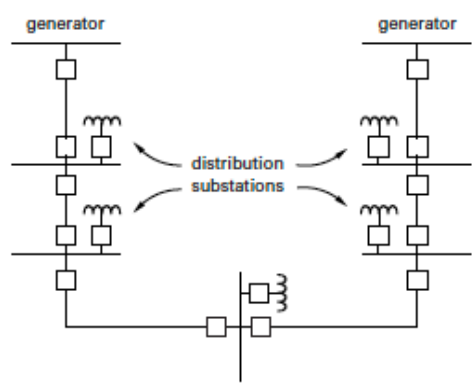

Figure 3: Loop Distribution Pattern

[3] The ANSI standard for Voltage Range A is 114/228 V to 126/ 252 V at the service entrance and 110/220 V to 126/252 V at the utilization point. See Ref [G] for further details.

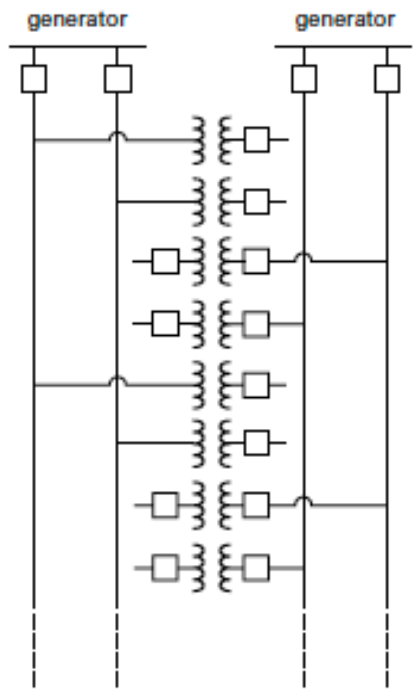

Figure 4: Multiple Distribution Pattern

COMMON-NEUTRAL SYSTEM

In practice, because of economic and operating advantages, almost all distribution systems are three-phase four-wire, common-neutral primary systems like the one shown in Fig. 5. The fourth wire acts as a neutral for both the primary and secondary systems (see Fig. 1) although one side of the transformer is delta. (The delta connection is used because the third-harmonic voltages generated by the nonlinear nature of the transformer circulate within the delta and do not affect the line currents or voltages.)

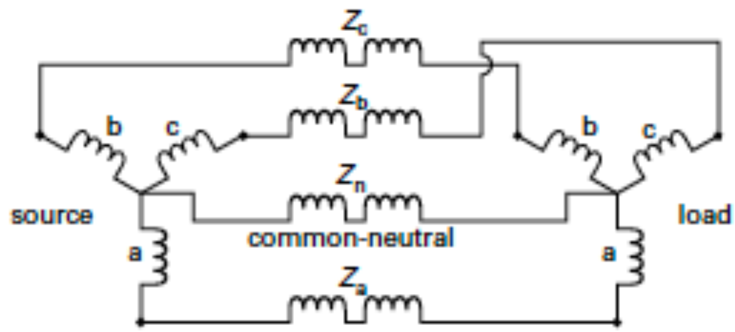

Figure 5: Common Neutral (Three-Phase Four-Wire) System

The neutral is grounded in numerous locations, including at each distribution transformer and to metallic water piping or grounding devices at each service entrance.[4] The fourth wire carries a portion of the unbalanced currents that may exist, while the remainder flows through the earth. Some of the advantages of such a system follow.

- Unbalanced currents flow through the neutral or the earth, thus ensuring approximately balanced three-phase current flow through the generator. This evens the counter torque, minimizes vibration, and extends bearing life.
- Single-phase circuits require only one insulated conductor and the uninsulated neutral, thus lowering wiring costs.
- Protective devices need to be placed on only one wire of a single-phase circuit to provide adequate protection, significantly lowering costs.

Other possible systems include the three-phase three-wire system, which is not widely used for public power distribution except in California. Such a system finds application in marine systems that operate ungrounded to improve reliability. Two-phase systems are rarely used. Direct-current systems, though they have some advantages, have been replaced by AC systems.[5] Series systems are used primarily for street lighting but have been largely replaced by multiple systems.

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[4] The use of nonmetallic water piping requires the use of other grounding techniques described in the National Electrical Code, Ref. [D].
[5] This is changing with some high-voltage DC systems in-use and being built.

OVERCURRENT PROTECTION

Overcurrent is current in excess of the rated current of the equipment or the ampacity of the conductors. It may be caused by an overload, short circuit, or ground fault. *Overload* is any condition beyond the rating of the electrical device supplied that if continued for an extended time will damage the equipment or cause overheating. A short circuit or ground fault is not an overload. The short circuit or ground fault conditions can result in extremely high currents for very short time periods. Overload conditions are longer term. Protective devices include relayed circuit breakers, reclosers, sectionalizers, and fuses.

A *circuit breaker* is an electromagnetic device used to open and close a circuit. The operation of a circuit breaker can be automatic or manual. For overcurrent protection, relays are installed that sense the current and create a corresponding magnetic field whose force is used to trip the breaker if necessary. The trip characteristics of some circuit breakers are adjustable. An *instantaneous trip* is one that occurs without delay. An *inverse time trip* is one in which a delay is deliberately instituted. As the current magnitude increases, the time to trip decreases. The *National Electrical Code* specifies that a circuit breaker is considered to provide adequate short-circuit protection if its rating is no more than six times the ampacity of the conductors.

A recloser is a device that opens a circuit instantaneously when a fault occurs but recloses after a short period. Reclosers typically have a continuous current rating of 560 A and an interrupting capacity of 16,000 A or less. A circuit breaker will typically have a continuous current rating of 1200 A and an interrupting capacity of 40,000 A under short-circuit conditions. A recloser normally operates at twice its current rating. This is called the *minimum pickup current*. Reclosers are used to protect the distribution system while minimizing outages.

A *sectionalizer* is a device that counts the consecutive number of times a recloser operates and opens the circuit at a predetermined number of counts, usually two or three. The sectionalizer has no interrupting capacity. It operates to keep the recloser open during the time that the recloser is open.

A *fuse* is an expendable device that opens a circuit when the current becomes excessive. The fuse is a sealed container that contains a conductor surrounded by quartz-sand filler. The conductor melts after the current exceeds a given value. Though normally used for overload conditions, special types of fuses called current-limiting fuses are designed to melt nearly instantaneously.

The *National Electrical Code* specifies that such fuses can be used as short-circuit protection if their rating does not exceed three times the ampacity of the conductors.

POLE LINES

The poles that support overhead electric utility lines are constructed of wood, concrete, steel, and aluminum. Poles are subject to *vertical loading* from the weight of the conducting cable. Normal *transverse loading* occurs when a pole is located at a corner, that is, a point where the geographic direction of the conducting cable changes. Additional vertical and transverse forces account for *wind loading* and *ice loading*.[6] This is accomplished by first calculating the forces using standard mechanical engineering techniques. Then the expected pressures on flat surfaces caused by the wind is determined from

Equation 1: Wind Loading Flat Surface

$$p = 71.43\text{v}^2$$

The term v represents the wind velocity normal to the flat surface in km/hr. The pressure is in units of Pa. For curved surfaces, such as a standard pole, the pressure is determined from

Equation 2: Wind Loading Curved Surface

$$p = 44.64\text{v}^2$$

Finally, a percentage of this force is added to the result of the mechanical and wind calculations to increase the amount of force that the pole must be designed to withstand. (The force calculations become moment calculations when the pole height is taken into consideration.) The percentage to be added is determined by the extreme wind and icing conditions expected in the area where the pole is to be located. The percentage may vary from 0.0 to 0.30.

Copper conductor used for overhead lines where spans are approximately 61 m (200 ft) or more is the harddrawn type due to its greater strength. For shorter spans, medium hard-drawn or annealed copper is utilized. For very long spans, aluminum stranded around a steel core is used. This type of cable is called aluminum conductor steel-reinforced (ACSR). High-strength aluminum alloys

[6] To be treated in more detail in Part V of this course.

are also used. One type is called aluminum conductor alloy-reinforced (ACAR). Another is the all-aluminum-alloy conductor (AAAC).

UNDERGROUND DISTRIBUTION

Underground installations have increased in popularity, especially in residential districts, primarily for aesthetic reasons, though there are indications that the frequency of faults is lower compared to overhead systems. When faults do occur, however, they are more difficult to access and repair. In a residential district, such a system is called an *underground residential distribution* (URD). Since the development of synthetic insulation such as polyethylene, aluminum conductor is used almost exclusively for underground installations.[7] In copper conductors, standard soft copper is used to improve flexibility. Unlike overhead lines, underground cables must be insulated to ensure the conductor does not contact ground or the cable's external sheath. The insulation also protects against mechanical, chemical, and other effects peculiar to an underground environment.

The insulating material surrounding a conductor is a dielectric. Figure 6(a) shows such an arrangement. The insulation thickness must be such that the electric field strength, E, at the surface of the conductor does not break down the insulation. If the cable is too large, it becomes difficult to handle and expensive. If the cable is too small, the dielectric loss becomes large, resulting in overheating of the cable. The optimal thickness is determined by the ratio in Eq. 3.

Equation 3: Underground Optimal Thickness

$$\frac{r_2}{r_1} = e = 2.718$$

Figure 6(a) illustrates a cable with a single dielectric. When more than one dielectric is present, they are arranged to minimize the difference between the maximum and minimum electric field strength across the cable. This arrangement is known as *grading*. *Capacitance grading* is illustrated in Fig. 6(b). Given the restriction of a maximum electric field, Emax, the operating voltage for a capacitance-graded cable is given by

[7] Aluminum conductor connections require special attention to ensure adequate contact and avoid corrosion.

Equation 4: Operating Voltage Capacitance Grading

$$V = E_{\max}\left(r_1 \ln\frac{r_2}{r_1} + r_2 \ln\frac{r_3}{r_2} \right)$$

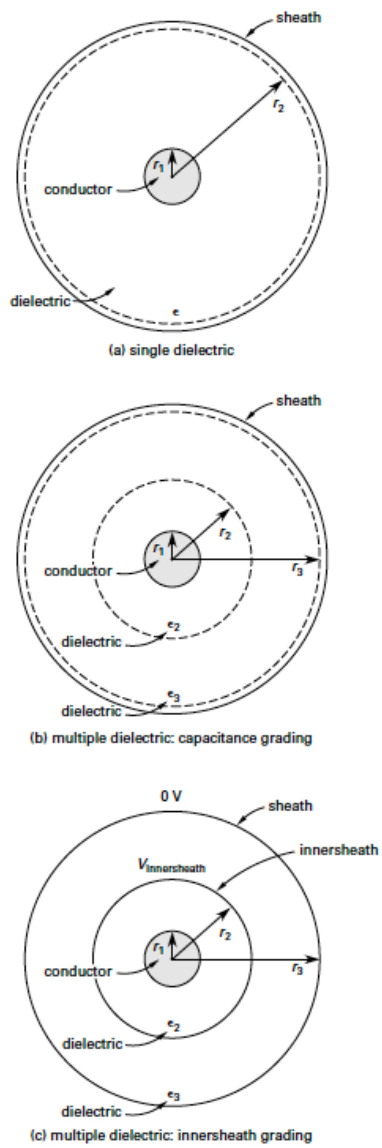

Figure 6: Insulated Conductors

When the dielectric material is separated by coaxial metallic sheaths maintained at a constant voltage level, the grading is called *innersheath grading* (see Fig. 6(c)). The sheathing is used to minimize the electric field. Let the ratio of the radii be

Equation 5: Innersheath Grading Ratio

$$\xi = \frac{r_3}{r_2} = \frac{r_2}{r_1}$$

The maximum electric field at the surface of the conductor is

Equation 6: Maximum Electric Field at Conductor Surface

$$E_{1,\text{max}} = \frac{V - V_{\text{innersheath}}}{r_1 \ln \frac{r_2}{r_1}} = \frac{V - V_{\text{innersheath}}}{r_1 \ln \xi}$$

The maximum electric field at the surface of the innersheath is

Equation 7: Maximum Electric Field at Innersheath Surface

$$E_{1,\text{max}} = \frac{V_{\text{innersheath}}}{r_2 \ln \frac{r_3}{r_2}} = \frac{V_{\text{innersheath}}}{r_2 \ln \xi}$$

The permittivity values are selected so that the maximum electric fields are the same. This condition results in the relationship given in Eq. 8 between the innersheath voltage and the operating voltage.

Equation 8: Innersheath Voltage

$$V_{\text{innersheath}} = \left(\frac{\xi}{1+\xi}\right)V$$

The capacitance per unit length, C_l, for a single-conductor cable is

Equation 9: Capacitance per Unit Length

$$C_l = \frac{Q}{V} = \frac{2\pi\varepsilon}{\ln\xi} \quad \text{[farads per meter]}$$

The inductance per unit length, L_l, for a single-conductor cable is

Equation 10: Inductance per Unit Length

$$L_l = \frac{\mu_0}{2\pi}\ln\xi \quad \text{[henries per meter]}$$

FAULT ANALYSIS: SYMMETRICAL

A fault is any defect in a circuit, such as an open circuit, short circuit, or ground. Short-circuit faults, called *shunt faults*, are shown in Fig. 7. Open-circuit faults are called series faults. Any fault that connects a circuit to ground is termed a *ground fault*. The balanced three-phase short circuit shown in Fig. 7(d) is one of the least likely, yet most severe, faults and thus determines the ratings of the supplying circuit breaker.

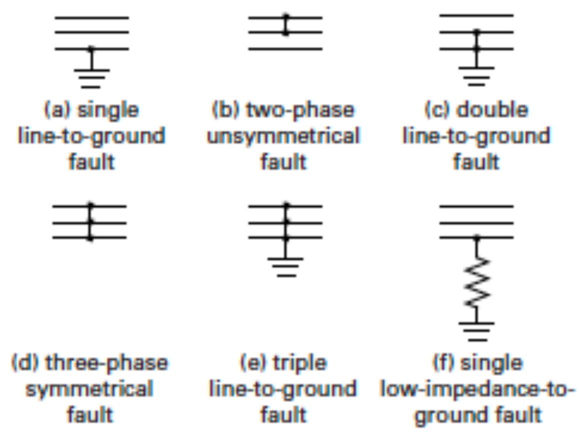

Figure 7: Fault Types, from Most Likely to Least Likely

A three-phase *symmetrical fault*, such as that in Fig. 7(d), has three specific time periods of concern, as shown in Fig. 8. During the *subtransient period*, which lasts only for a few cycles, the current rapidly decreases and the synchronous reactance, X_s, changes to the subtransient reactance, X''_d.[8] The model for a synchronous generator changes from that in Fig. 9(a) to that in 9(b). The sudden change in armature current results in a lower armature reactance, but the current through the leakage reactance must remain the same for continuity of energy. Thus, for proper modeling, it is necessary to change the voltage, E_g, to E''_g. The voltage E''_g is calculated for the subtransient interval *just prior to the initiation of the fault using Eq. 11.*[9]

Equation 11: Subtransient Voltage

$$E''_g = V_t + jI_L X''_d$$

The subtransient time period lasts a few cycles and is defined in the IEEE standards as the time it takes for the rapidly changing component of the direct-axis, short-circuit current to decrease to 1/e, or 0.368, of its initial value.[10]

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[8] There are actually two reactances: the *direct reactance* that lags the generator voltage by 90°, and a *quadrature reactance* (X_q) that is in phase with the generator voltage. During faults, the power factor is low and the quadrature reactance can be ignored.

[9] The general form for the synchronous generator voltage is $E''_g = V_t + jIX$.

[10] Subtransient times vary with system electrical values, but "a few" cycles is often the range: approximately 0 cycles to 4 cycles, or 0 ms to 66 ms. Transient times last approximately 6 cycles, or from 100 ms to as long as 5 s.

The transient time period is defined in the standards as the time it takes for the slowly changing component of the direct-axis, short-circuit current to decrease to 1/e, or 0.368, of its initial value.

During the *transient period*, a similar situation exists and the model for the synchronous generator changes from that in Fig. 9(b) to that in 9(c). The correct generator voltage for this period is calculated *just prior to the initiation of the fault using Eq. 12.*

Equation 12: Transient Voltage

$$E_g' = V_t + jI_L X_d'$$

Both E_g'' and E_g' depend on the impedance of the load and the resulting current prior to fault initiation.

Typical reactances of three-phase synchronous machines are given in Table 1. When a synchronous motor is part of a system, it is considered a synchronous generator for fault analysis.[11]

[11] The motor actually becomes a generator when a fault occurs. The spinning motor changes the mechanical energy it possesses into electrical energy, supplying the fault and slowing in the process.

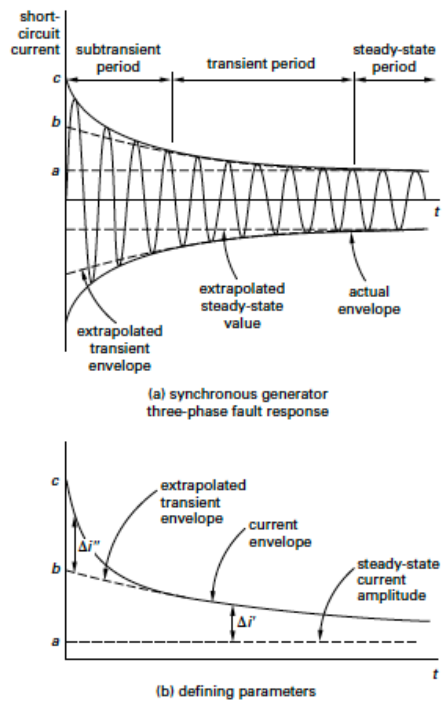
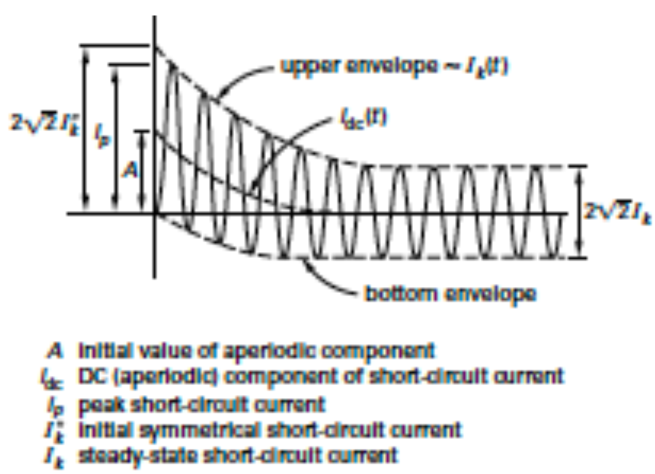


Figure 8: Symmetrical Fault Terminology

IEEE defines each of the periods as shown below, which is consistent with Fig. 8. Note the square root of 2 is to obtain the rms values from the peak values.



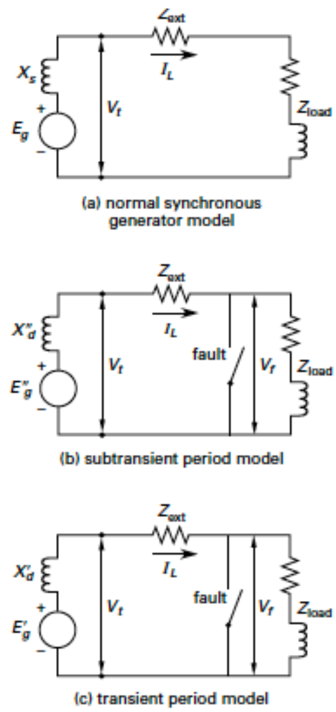


Figure 9: Synchronous Generator Fault Models

Table 1: Typical Reactances of Three-Phase Synchronous Machines

[Typical values are given above the bars. The ranges are below the bars. Always use the latest values available.]

	X_d (unsaturated)	X_q (rated current)	X'_d (rated voltage)	X''_d (rated voltage)
two-pole turbine generators	1.20 / 0.95–1.45	1.16 / 0.92–1.42	0.15 / 0.12–0.21	0.09 / 0.07–0.14
four-pole turbine generators	1.20 / 1.00–1.45	1.16 / 0.92–1.42	0.23 / 0.20–0.28	0.14 / 0.12–0.17
salient-pole generators and motors (with dampers)	1.25 / 0.60–1.50	0.70 / 0.40–0.80	0.30 [a] / 0.20–0.50	0.20 [a] / 0.13–0.32
salient-pole generators (without dampers)	1.25 / 0.60–1.50	0.70 / 0.40–0.80	0.30 [a] / 0.20–0.50	0.30 [a] / 0.20–0.50
capacitors (air-cooled)	1.85 / 1.25–2.20	1.15 / 0.95–1.30	0.40 / 0.36–0.50	0.27 / 0.19–0.30
capacitors (hydrogen-cooled at 1/2 psi)	2.20 / 1.50–2.65	1.35 / 1.10–1.55	0.48 / 0.36–0.60	0.32 / 0.23–0.36

[a]High-speed units tend to have low reactance, and low-speed units tend to have high reactance.

Used with permission from *Electrical Transmission and Distribution Reference Book*, by permission of the Westinghouse Electric Corporation.

The steady-state period lasts from the end of the transient period until the circuit breaker or other interrupting device acts to remove the fault. Circuit breakers must be able to interrupt fault currents at the rated voltage. The usual fault ratings are in kVA or MVA. The fault rating is the product of the interrupt current capacity and the line-to-line kV rating.

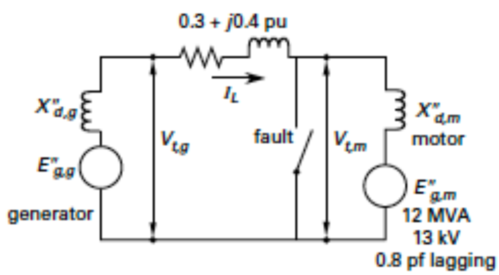
Example 1

A synchronous generator and motor are rated for 15 MVA, 13.9 kV, and 25% subtransient reactance. The line impedance connecting the generator and motor is $0.3 + j0.4$ pu, given with the ratings as the base. The motor is operating at 12 MVA, 13 kV, with a 0.8 pf lagging.

What is the per-unit equivalent generated voltage for the motor, $E''_{g,m}$, with the expectation of a fault at the motor terminals?

Solution

The one-line diagram illustrating the situation is as follows.



Determine the base quantities for this three-phase system.

$$S_{\text{base}} = S_t = 15 \text{ MVA}$$

$$V_{\text{base}} = V_t = 13.9 \text{ kV}$$

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{15\times 10^6 \text{ VA}}{\sqrt{3}\left(13.9\times 10^3 \text{ V}\right)} = 0.62 \text{ kA}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{\sqrt{3}\,I_{\text{base}}} = \frac{13.9 \text{ kV}}{\sqrt{3}\left(0.62 \text{ kA}\right)} = 12.9\ \Omega$$

Determine the per-unit quantities of interest, that is, the voltage at the fault site (the terminals of the motor) and the current flowing.

$$V_{\text{pu}} = \frac{V_{\text{actual}}}{V_{\text{base}}} = \frac{13.0 \text{ kV}}{13.9 \text{ kV}} = 0.935 \text{ pu}$$

$$I_{\text{pu}} = \frac{I_{\text{actual}}}{I_{\text{base}}} = \frac{S/V}{I_{\text{base}}} = \frac{\left(12\times10^6 \text{ VA}\right)/\left(\sqrt{3}\left(13\times10^3 \text{ V}\right)\right)}{0.62\times10^3 \text{ A}} = 0.860$$

The power factor is 0.8 lagging, thus

$$\text{lagging pf} = -\cos\theta = -0.8$$
$$\theta = -\cos^{-1}0.8 = -36.9^\circ$$

The line current, I_l, is

$$\begin{aligned} I_l &= I_{\text{pu}}\angle\theta = 0.860 \text{ pu}\angle -36.9^\circ \\ &= \left(\cos\theta\right)\left(0.860 \text{ pu}\right) + j\left(\sin\theta\right)\left(0.860 \text{ pu}\right) \\ &= 0.688 - j0.516 \text{ pu} \end{aligned}$$

The equivalent generated voltage for the motor can now be found. Writing the equation for the voltage from the motor terminals to the motor armature gives

$$\begin{aligned} E''_{g,m} &= V_{t,m} - jX''_d I_t \\ &= 0.935 - j\left(0.25\right)\left(0.688 - j0.516\right) \\ &= 0.935 - \left[\left(j0.172\right) - \left(j^2\right)\left(0.25\right)\left(0.516\right)\right] \\ &= 0.806 - j0.172 \\ &= 0.824 \text{ pu}\angle 12^\circ \end{aligned}$$

Example 2

Determine the per-unit equivalent generated voltage for the generator, $E''_{g,g}$, for the system in Example 1.

Solution

The motor terminal voltage is known and the line current is known. An equation for the equivalent generated voltage, written from the known quantities at the motor terminals, is

$$E''_{g,g} = V_{t,m} + I_l Z$$

The impedance from the motor terminal back to the generator, which is the only unknown. Thus,

$$\begin{aligned} Z &= Z_l + Z_g \\ &= (0.3 + j0.4) + j0.25 \\ &= 0.3 + j0.65 \\ &= 0.715 \text{ pu}\angle 65^\circ \end{aligned}$$

Substituting gives

$$\begin{aligned} E''_{g,g} &= V_{t,m} + I_l Z \\ &= 0.935 + (0.860\angle -36.9^\circ)(0.715\angle 65^\circ) \\ &= 1.5 \text{ pu}\angle 11.1^\circ \end{aligned}$$

Example 3

Determine the per-unit substransient period current for the motor in Example 1.

Solution

A fault at the motor terminals results in current given by

$$I_m = \frac{E''_{g,m}}{jX''_d} = \frac{0.824 \text{ pu}\angle -12^\circ}{0.25 \text{ pu}\angle 90^\circ} = 3.3 \text{ pu}\angle -102^\circ$$

Example 4

Determine the per-unit subtransient period current for the generator in Example 1.

Solution

The generator current that flows to the fault is given by

$$I_g = \frac{E''_{g,g}}{Z} = \frac{1.5 \text{ pu}\angle 11.1^\circ}{0.715 \text{ pu}\angle 65^\circ} = 2.09 \text{ pu}\angle -53.9^\circ$$

Example 5

What is the actual fault current for the circuit in Example 1?

Solution

The actual fault current is the total current through the fault from the generator and the motor. The synchronous motor acts as a generator to the fault as it gradually loses speed. *The subtransient period is too short for the motor to lose a significant amount of speed. Thus, it can be treated as a generator.* The total current is

$$\begin{aligned} I &= \mathrm{I}_m + I_g \\ &= 3.3 \text{ pu}\angle -102^\circ + 20.9 \text{ pu}\angle -53.9^\circ \\ &= 4.95 \text{ pu}\angle -83.7^\circ \end{aligned}$$

The actual current is

$$\begin{aligned} I_{\text{actual}} &= I_{\text{pu}} I_{\text{base}} \\ &= \left(4.95 \text{ pu}\angle -83.7^\circ\right)\left(0.62 \text{ kA}\right) \\ &= 3.07 \text{ kA}\angle -83.7^\circ \end{aligned}$$

Example 6

Determine the minimum rating necessary for the motor and generator circuit breakers to protect against the fault described in Example 1.

Solution

The minimum rating is determined by the per-unit current drawn during the fault and the base apparent power, that is, the base VA. In other words, the rating is the base apparent power (VA_{base}) multiplied by a factor representing the current flowing above the base current.

$$\text{VA}_{\text{rating}} = I_{\text{pu,fault}}\text{VA}_{base}$$

For the motor circuit breaker,

$$\begin{aligned}\text{VA}_{\text{rating},m} &= I_{\text{pu,fault}}\text{VA}_{base} \\ &= (3.3)(15\ \text{MVA}) \\ &= 49.5\ \text{MVA}\end{aligned}$$

For the generator circuit breaker,

$$\begin{aligned}\text{VA}_{\text{rating},\,g} &= I_{\text{pu,fault}}\text{VA}_{\text{base}} \\ &= (2.09)(15\ \text{MVA}) \\ &= 31.4\ \text{MVA}\end{aligned}$$

FAULT ANALYSIS: UNSYMMETRICAL

Unsymmetrical faults, also called *asymmetrical faults*, are any faults other than a three-phase short. Examples include the line-to-line and line-to-ground faults shown in Figs. 7(b) through 7(f). Such faults are more common than three-phase shorts, but they are more difficult to analyze because they result in uneven phase voltages and currents. Nevertheless, unsymmetrical faults can be analyzed by separating unbalanced phasor components into three sets of symmetrical components as shown in Fig. 10.[12]

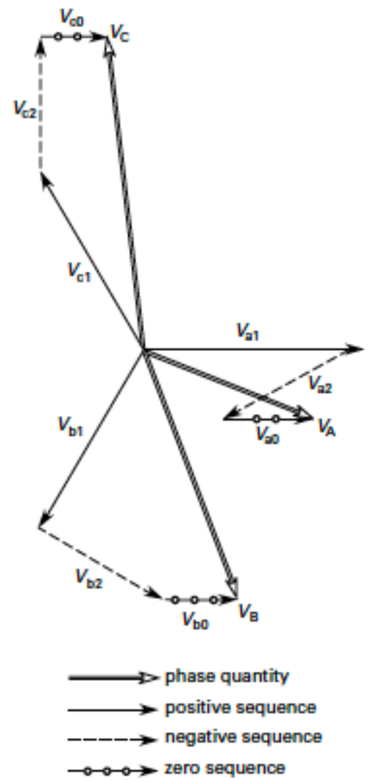


Figure 10: Phasor Diagram: Symmetrical Components of Unbalanced Phasors

The three sets of symmetrical components shown in Fig. 35.10 are referred to as the *positive-sequence*, *negative-sequence*, and *zero-sequence* components of the unsymmetrical phasors. The positive-sequence phasors, shown again in Fig. 11(a), are three equal-magnitude phasors rotating counterclockwise in sequence a, b, c. Positive-sequence phasors are 120 electrical degrees apart and sum to zero. Positive-sequence components are the ones normally used in electrical engineering. Such components represent balanced three-phase generators, motors, and transformers. The subscript 1 is normally used to indicate the positive sequence.

The negative-sequence phasors, shown again in Fig. 11(b), are three equal-magnitude phasors rotating counterclockwise in sequence a, c, b. They can also be represented as a mirror image of the positive-sequence phasors rotating in the clockwise direction. Negative sequence phasors are

[12] For more detailed analysis of power systems, see Ref. [G].

120 electrical degrees apart and sum to zero. The negative-sequence reactance is the average of the subtransient direct and quadrature reactances, as given in Eq. 13. The subscript 2 is normally used to indicate the negative sequence.

Equation 13: Negative Sequence Reactance

$$X_2 = \frac{1}{2}\left(X_d'' + X_q''\right)$$

The zero-sequence phasors, shown again in Fig. 11(c), are three equal-magnitude phasors coincident in phase sequence and rotating counterclockwise. The subscript 0 is normally used to indicate the zero sequence.

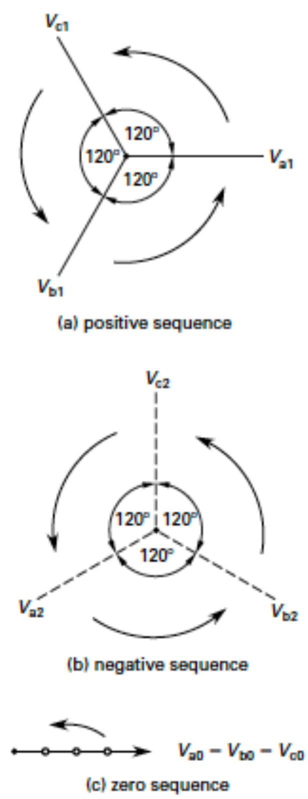


Figure 11: Components of Unsymmetrical Phasors

The unsymmetrical phasors of Fig. 10 are represented in terms of their symmetrical components by the following equations.

Equation 14: Unsymmetrical Phasor A in Symmetrical Components

$$V_{\mathrm{A}} = V_{\mathrm{a0}} + V_{\mathrm{a1}} + V_{\mathrm{a2}}$$

Equation 15: Unsymmetrical Phasor B in Symmetrical Components

$$V_{\mathrm{B}} = V_{\mathrm{b0}} + V_{\mathrm{b1}} + V_{\mathrm{b2}}$$

Equation 16: Unsymmetrical Phasor C in Symmetrical Components

$$V_{\mathrm{C}} = V_{\mathrm{c0}} + V_{\mathrm{c1}} + V_{\mathrm{c2}}$$

Consider an operator a defined as a unit vector with a magnitude of one and an angle of 120°, or the quantity $1\angle 120^\circ$. (This is similar to the operator j of $1\angle 90^\circ$.) The properties of such an operator are as follows.[13]

Equation 17: Operator a

$$a = 1\angle 120^\circ = 1 \times e^{j120^\circ} = -0.5 + j0.866$$

Equation 18: Operator a*

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866 = a^*$$

Equation 19: Operator a³

$$a^3 = 1\angle 360^\circ = 1\angle 0^\circ$$

Equation 20: Operator a⁴

$$a^4 = a$$

[13] Just as **j** is not normally written in vector notation, the operator **a** is normally written as “a.”

Equation 21: Operator a^5

$$a^5 = a^2$$

Equation 22: Operator a^6

$$a^6 = a^3$$

Equation 23: Operator a Summation

$$1 + a + a^2 = 0$$

Using Eqs. 14 through 16 and the concept of the phasor a in Eqs. 17 through 23, the unsymmetrical components can be represented in terms of a single phase. For example, using phase A gives

Equation 24: Unsymmetrical Phasor A Using Operator a

$$V_{\mathrm{A}} = V_{\mathrm{a0}} + V_{\mathrm{a1}} + V_{\mathrm{a2}}$$

Equation 25: Unsymmetrical Phasor B Using Operator a

$$V_{\mathrm{B}} = V_{\mathrm{a0}} + a^2 V_{\mathrm{a1}} + a V_{\mathrm{a2}}$$

Equation 26: Unsymmetrical Phasor C Using Operator a

$$V_{\mathrm{C}} = V_{\mathrm{a0}} + a V_{\mathrm{a1}} + a^2 V_{\mathrm{a2}}$$

Solving for the sequence components from Eqs. 24 through 26 gives

Equation 27: Zero Sequence Components

$$V_{\mathrm{a0}} = \frac{1}{3}\left(V_{\mathrm{A}} + V_{\mathrm{B}} + V_{\mathrm{C}}\right)$$

Equation 28: Positive Sequence Components

$$V_{\mathrm{a1}} = \frac{1}{3}\left(V_{\mathrm{A}} + a V_{\mathrm{B}} + a^2 V_{\mathrm{C}}\right)$$

Equation 29: Negative Sequence

$$V_{\mathrm{a2}} = \frac{1}{3}\left(V_{\mathrm{A}} + a^2 V_{\mathrm{B}} + a V_{\mathrm{C}}\right)$$

Equations similar to Eqs. 24 through 29 exist for currents as well. Thus, sequence impedances may also be defined. Impedance through which only positive-sequence currents flow is called a *positive-sequence impedance*. Positive-sequence networks are normally used in electrical engineering and an example is given in Fig. 12(b). *Negative-sequence impedance* is similar to positive-sequence impedance with the reactance given by Eq. 13. In addition, a negative-sequence network omits all positive-sequence generators. A sample negative-sequence network is shown in Fig. 12(c). *Zero-sequence impedance* is significantly different from positive-sequence impedance. The only machine impedance seen by the zero-sequence impedance is the leakage reactance, X_0. Series reactance is greater than positive-sequence reactance by a factor of 2 to 3.5. A sample zero-sequence network is shown in Fig. 12(d). Only a wye-connected load with a grounded neutral permits zero-sequence currents. Only a delta-connected transformer secondary permits zero-sequence currents. Figure 13 shows zero-sequence impedances for various configurations. The use of sequence networks simplifies fault calculations.

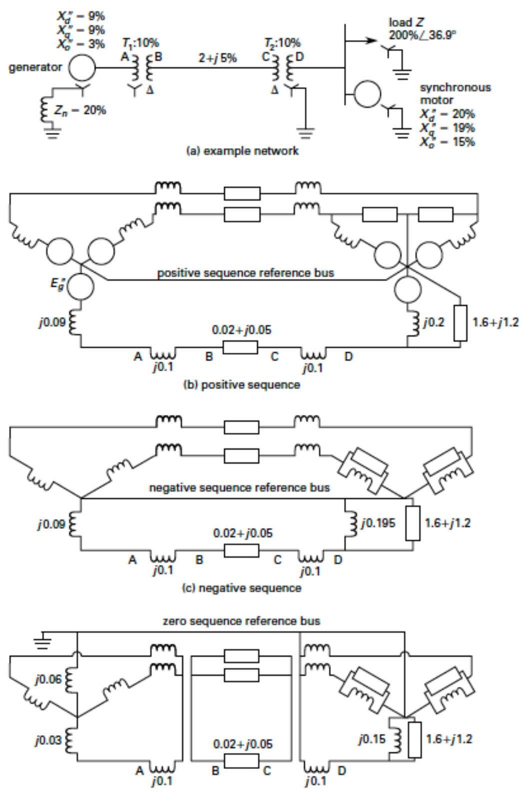


Figure 12: Sample Sequence Networks

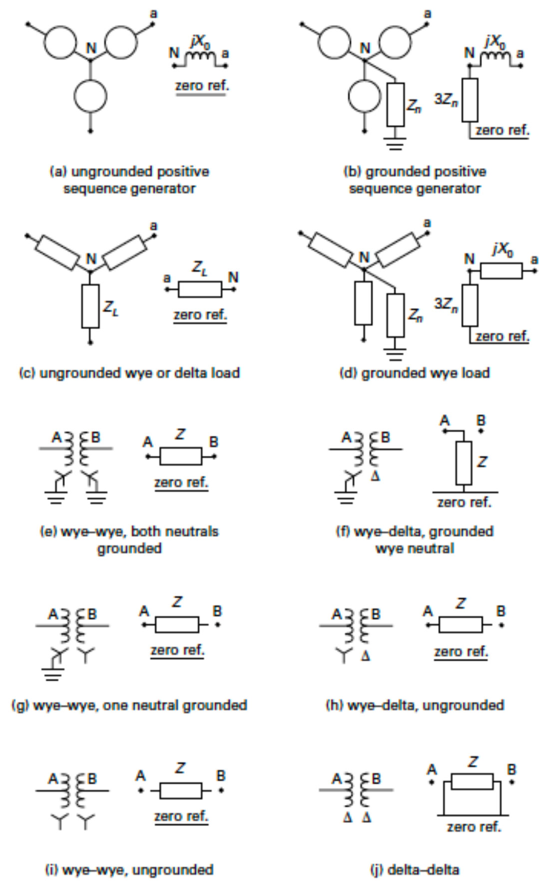


Figure 13: Zero Sequence Impedances

FAULT ANALYSIS: MVA METHOD

The fault analysis techniques explained earlier provided detailed information on the fault, and *require significant calculation time*. Information on those techniques may be found in IEEE Standard 141-1993 (revised in 1996) titled *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants.* International standard IEC-909 also provides guidance for short-circuit analysis that differs from the IEEE guidance, and in some ways is more detailed. Numerous methods exist. The names of a few methods are *ohmic*, *Thevenin*, *E/Z*, and *E/X*. Software exists to implement these short-circuit analysis methods.

A simplified method, which nevertheless captures the worst-case fault current, is called the *MVA method*. The MVA method can be determined without significant calculation time or software support. This method calculates the fault current that will flow in a system or component for unit voltage supplied from an infinite source. (The infinite source assumption means that the power sources are treated as *reactors*, that is, inductive components with no resistance, and the voltage is steady at the base value throughout.) The MVA method uses the per-unit system, maintains the voltage at a 1 pu value, and calculates the fault (or short circuit) apparent power, S_{fault} or S_{sc}, and the fault (or short circuit) current, I_{fault} or I_{sc}. The equations for the maximum fault power and fault current are

Equation 30: Maximum Fault Power

$$S_{sc} = \frac{S_{\text{base}}}{Z_{pu}}$$

Equation 31: Short-Circuit Current

$$I_{sc} = \frac{S_{sc}}{\sqrt{3}V_{\text{base}}}$$

Variations of this method are used, including one that adds the power sources (i.e., the MVA sources) as vectors. Series power sources are treated as capacitors in series, given that the power transferred to the fault must go through both sources. For example, a generator has to pass its power through an intervening transformer.[14] Parallel power sources are treated as capacitors in parallel, given that power from both is available to the fault.

Motors are treated as generators during short circuit fault analysis because rotational energy is converted to electrical energy and transferred to the fault during such events. Standard practices vary as to the contributions from various size motors.

Finally, the equations used here assume symmetrical faults. The power may be calculated based on the positive, negative, and zero sequence impedances, and then combined to provide results for unsymmetrical faults.

Explanatory examples follow.

[14] Which then limits the power to that which the transformer can supply.

Example 7

Derive the equation for the fault apparent power, S_{sc}, Eq. 30.[15]

Solution

The base apparent power is as follows.

$$S_{\text{base}} = \sqrt{3}\, I_{\text{base}} V_{\text{base}} \qquad \text{[I]}$$

Rearrange, solve for the base current.

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}\, V_{\text{base}}} \qquad \text{[II]}$$

Now, the fault apparent power and current will use the base voltage as this provides the worst-case condition, as well as allowing for per-unit calculations across transformers. Thus, changing Eqs. I and II to their fault versions gives the following.

$$S_{sc} = \sqrt{3}\, I_{sc} V_{\text{base}} \qquad \text{[III]}$$

$$I_{sc} = \frac{S_{sc}}{\sqrt{3}\, V_{\text{base}}} \qquad \text{[IV]}$$

Equation IV is in the form of Eq. 31. Now, the short circuit (fault) current, I_{sc}, is the actual current that flows in the fault. Using the per-unit system, this gives

$$I_{sc} = I_{\text{actual}} = I_{\text{base}} I_{\text{pu}} \qquad \text{[V]}$$

[15] Derivations need not be remembered except for one item. Once one understands the derivation and the subsequent result as truth, this can be accepted and used in future problems.

The per-unit current is given by

$$I_{\text{pu}} = \frac{V_{\text{pu}}}{Z_{\text{pu}}} \qquad \text{[VI]}$$

Noting the per-unit value of the voltage is 1 pu, given that the base voltage is being used, changes Eq. VI into

$$I_{\text{pu}} = \frac{1}{Z_{\text{pu}}} \qquad \text{[VII]}$$

Substituting into Eq. V gives

$$I_{sc} = I_{\text{actual}} = I_{\text{base}}\left(\frac{1}{Z_{\text{pu}}}\right) \qquad \text{[VIII]}$$

Substituting Eq. VIII into Eq. III gives

$$\begin{aligned} S_{sc} &= \sqrt{3}\left(I_{\text{base}}\left(\frac{1}{Z_{\text{pu}}}\right)\right)V_{\text{base}} \\ &= \frac{\sqrt{3}\,I_{\text{base}}V_{\text{base}}}{Z_{\text{pu}}} \qquad \text{[IX]} \end{aligned}$$

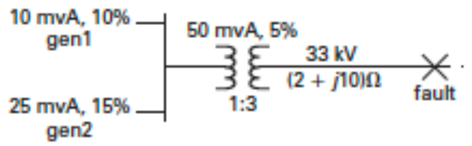
Noting that the numerator in Eq. IX is the formula for the base apparent power gives

$$S_{sc} = \frac{\sqrt{3}\,I_{\text{base}}V_{\text{base}}}{Z_{\text{pu}}} = \frac{S_{\text{base}}}{Z_{\text{pu}}} \qquad \text{[X]}$$

Equation [X] is Eq. 30, and completes the derivation. Note: because of the assumptions, this is the maximum fault value expected. (Refer to Eqs. 30 and 31.)

Example 8

Consider the one-line diagram for a three-phase generator-reactor system in the illustration. The power ratings and per-unit impedances based on those ratings are given. Assume a worse-case symmetrical fault.



Using the MVA method, what is the maximum short circuit current at the fault location shown?

Solutions

Assume an arbitrary base of 100 MVA (since multiple bases are involved).[16]

Using an arbitrary base of 100 MVA for S_{base} and 33 kV for V_{base} [always use the base voltage in the area of the fault], Eq. [I.a] below converts to the following reactance values from the given rating base values to the new base.[17]

(The reactances, or reactors, are the source of energy to the fault.) A base voltage of 33 kV was used to ensure the proper per-unit value was calculated for the line impedance because the conversion of actual to per-unit values must use the voltage present in the given section of the distribution system. Additionally, the fault is in the 33 kV section of the distribution system, so using 33 kV ensures the correct current calculation for the fault current. Reactance values, X, are used instead of impedance values, Z, because the sources are considered infinite, that is, no resistance is present.

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[16] The base itself doesn't matter in that each generator and transformer actuals per unit values are used, with the correct voltage and impedance. See Eqs. [II] through [IV].

[17] Equation [I.a] may seem incorrect; that is, upside down—it is not. In addition, converting the per unit impedance is a bit more involved. Equation [I.b] is shown for clarification. See Ref. [B] Chapter 34. Eqs. 34.53 and 34.54, for a detailed discussion.

The results of the calculations are

$$\chi_{\text{pu,new}} = \chi_{\text{pu,old}}\left(\frac{\chi_{\text{base,old}}}{\chi_{\text{base,new}}}\right) \qquad \text{[I.a]}$$

$$Z_{\text{pu,new}} = Z_{\text{pu,old}}\left(\frac{V_{\text{base,old}}}{V_{\text{base,new}}}\right)^2\left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}}\right) \qquad \text{[I.b]}$$

$$\mathbf{Z}_{\text{pu,gen1}} = jX_{\text{gen1}} = j0.10\left(\frac{100\text{ MVA}}{10\text{ MVA}}\right) = j1.0\text{ pu} \qquad \text{[II]}$$

$$\mathbf{Z}_{\text{pu,gen2}} = jX_{\text{gen2}} = j0.15\left(\frac{100\text{ MVA}}{25\text{ MVA}}\right) = j0.6\text{ pu} \qquad \text{[III]}$$

$$\mathbf{Z}_{\text{pu,transformer}} = jX_{\text{transformer}} = j0.05\left(\frac{100\text{ MVA}}{50\text{ MVA}}\right) = j0.1\text{ pu} \qquad \text{[IV]}$$

In some texts, only the reactance, *X*, is shown. Even so, when used in per-unit calculations, the *j* is added as a reminder that the value is for a reactive component, and must be added as a vector to any resistive components. Additionally, the impedance, **Z**, may not be shown in the bold format, even though it carries angular information. The form used in Eqs. I through IV is in keeping with IEEE standards (See Ref.

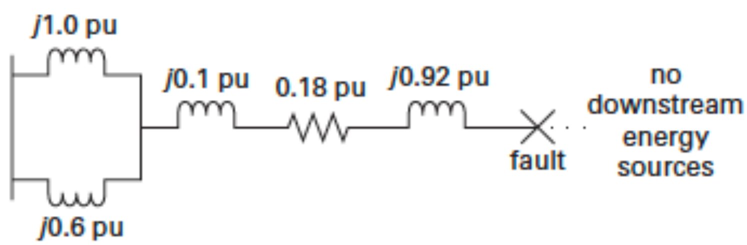
The impedance of the line is calculated as follows.

$$\mathbf{Z}_{\text{pu}} = \frac{\mathbf{Z}_{\text{actual}}}{Z_{\text{base}}} = \frac{\mathbf{Z}_{\text{actual}}}{\left(\frac{V_{\text{base}}^2}{S_{\text{base}}}\right)} = \mathbf{Z}_{\text{actual}}\left(\frac{S_{\text{base}}}{V_{\text{base}}^2}\right)$$

$$= \left(2\ \Omega + j10\ \Omega\right)\left(\frac{100\times 10^6\ \text{VA}}{\left(33\times 10^3\ \text{V}\right)^2}\right)$$

$$= \left(0.18 + j0.92\right)\ \text{pu}$$

It is important to note that the per-unit values of *reactance and impedance do NOT change as one considers different portions of the distribution system, that is, on either side of the transformer.*[18] However, the base voltages do change. *The correct base voltage must be used in calculating the per-unit values.* Once complete, the per-unit values may be combined in any way desired, as shown in the next several steps. The results of Eqs. I through V are as shown in the following illustration.



Combining the generator reactances gives

$$Z_{\text{total,gen}} = jX_{\text{total,gen}}$$

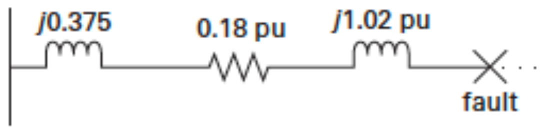
$$= \frac{\left(jX_{\text{gen1}}\right)\left(jX_{\text{gen2}}\right)}{jX_{\text{gen1}} + jX_{\text{gen2}}}$$

$$= \frac{\left(j1.0\ \text{pu}\right)\left(j0.6\ \text{pu}\right)}{j1.0\ \text{pu} + j0.6\ \text{pu}}$$

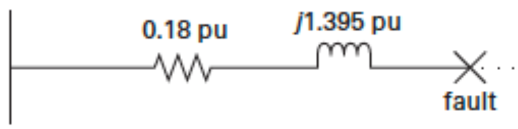
$$= j0.375\ \text{pu} \qquad \text{[VI]}$$

[18] See Ref. [B] Chapter 34 for a detailed explanation. This concept is the reason one can use a one-line diagram as shown.

Showing the results of Eq. [VI], and combining the transformer and line reactances gives



Combining the remaining reactances gives



The total impedance is

$$\mathbf{Z}_{\text{total}} = \left(0.18 + j1.395)\text{pu}\right) = 1.41\ \text{pu}\angle 86^{\circ} \qquad [\text{VII}]$$

The angle does not determine the maximum short circuit current, but it is required for phase calculations, and is shown here as clarification only. All the per unit quantities carry an angle that may be determined if necessary. Use Eq. 30 and the result of Eq. [VII] to calculate the short-circuit power.

$$S_{sc} = \frac{S_{\text{base}}}{Z_{\text{pu}}} = \frac{100\ \text{MVA}}{1.41\ \text{pu}} = 70.9\ \text{MVA} \qquad [\text{VIII}]$$

Substitute the result of Eq. [VIII] into Eq. 31 to determine the fault current at the indicated location.

$$I_{sc} = \frac{S_{sc}}{\sqrt{3}V_{\text{base}}} = \frac{70.9\times 10^{6}\ \text{VA}}{\sqrt{3}\left(33\times 10^{3}\ \text{V}\right)} = 1240.4 \quad \left(1240\ \text{A}\right) \qquad [\text{IX}]$$

SMART GRID

Though there is no single definition of a *smart grid*, the term generally applies to broad-based electrical distribution systems using some form of computer-based remote control and automation. The “grid" connects power generation to consumers using substations, transformers, switches, wires, communication infrastructure, and other elements. The “smart" components monitor elements of the system, predict electrical demand, and respond to load deviations and transients to provide continuous power.

The properties of smart grids are guided by Title XIII, “Smart Grid,” of the Energy Independence and Security Act of 2007 (EISA), which classifies the following types of projects as smart grid projects.

• projects that optimize the monitoring and control of transmission and distribution, including the use of sensors, communication links, and computer software and systems

• communication infrastructure projects to support the grid

• projects incorporating renewables

• microgrid projects supporting highly reliable and resilient *islanded operation* (i.e., smaller grids operating independently of the main grid or through a single interface with that grid)

• automation projects designed to increase information technology, communications, and overall cyber security

IEEE 3000 STANDARDS COLLECTION

The IEEE has established standards to guide the production, distribution, and utilization of electric energy. These IEEE standards are collectively known as the *IEEE 3000 Standards Collection™ for Industrial and Commercial Power Systems*, formerly as the *IEEE Color Books*. Each book focuses on one aspect of electric power and provides a basis for assessing the requirements for a given electrical project.

• IEEE Standard 141, *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants* (IEEE Red Book).

• IEEE Standard 142, *IEEE Recommended Practice for Grounding of Industrial and Commercial Power Systems* (IEEE Green Book).

• IEEE Standard 241, *IEEE Recommended Practice for Electric Power Systems in Commercial Buildings* (IEEE Gray Book).

• IEEE Standard 242*, IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems* (IEEE Buff Book).

• IEEE Standard 399, *IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis* (IEEE Brown Book).

• IEEE Standard 446, *IEEE Recommended Practice for Emergency and Standby Power Systems for Industrial and Commercial Applications* (IEEE Orange Book).

• IEEE Standard 493, *IEEE Recommended Practice for the Design of Reliable Industrial and Commercial Power Systems* (IEEE Gold Book).

• IEEE Standard 551, *IEEE Recommended Practice for Calculating AC Short-Circuit Currents in Industrial and Commercial Power Systems* (IEEE Violet Book).

• IEEE Standard 602, *IEEE Recommended Practice for Electric Systems in Health Care Facilities* (IEEE White Book).

• IEEE Standard 739, *IEEE Recommended Practice for Energy Management in Industrial and Commercial Facilities* (IEEE Bronze Book).

• IEEE Standard 902, *IEEE Guide for Maintenance, Operation, and Safety of Industrial and Commercial Power Systems* (IEEE Yellow Book).

• IEEE Standard 1015, *IEEE Recommended Practice for Applying Low-Voltage Circuit Breakers Used in Industrial and Commercial Power Systems* (IEEE Blue Book).

• IEEE Standard 1100, *IEEE Recommended Practice for Powering and Grounding Electronic Equipment* (IEEE Emerald Book).

IEEE RED BOOK

IEEE Standard 141, *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants* (*IEEE Red Book*) compiles the best practices for the design of electric systems for industrial plants, facilities, and associated buildings. It contains information often extracted from other codes, standards, and other technical literature. Chapters cover planning, voltage considerations, short-circuit calculations, protective devices, surge protection, power factors, harmonics, power switching, instrumentation, cable systems, busways, energy management, interface considerations, cost, and power system device numbering.

IEEE GRAY BOOK

IEEE Standard 241, *IEEE Recommended Practice for Electric Power Systems in Commercial Buildings* (*IEEE Gray Book*) describes the best practices for the electrical design of commercial buildings. It contains information often extracted from other codes, standards, and other technical literature on the requirements of such structures. Chapters cover load characteristics, voltage considerations, power sources, distribution apparatuses, controllers, services, and equipment vaults. The IEEE Gray Book also contains information on electrical equipment rooms, wiring, protection, lighting, space conditioning, automation, expansion, modernization, special occupancy requirements, and energy management.

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$\text{radians}/(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

Appendix B: Physical Constants

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N_A
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^5 Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
Velocity[3]			
Earth escape		3.67×10^4 ft/sec	1.12×10^4 m/s
light (vacuum)	c, c_0	9.84×10^8 ft/sec	2.9979 (3.00) $\times 10^8$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]	a	1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/°R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π	ħ		1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm-°R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02°F, 0.0888 psia	0.01109°C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol-°R 1.986 BTU/lbmol-°R	8314 J/kmol·K

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Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.

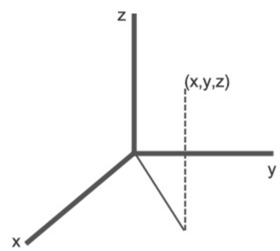
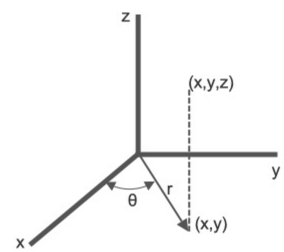
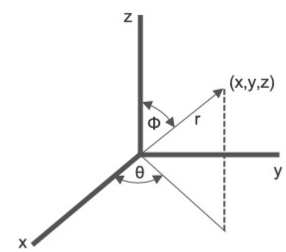
Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	pmega

Appendix F: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2 \sin\theta}\mathbf{r} & \frac{1}{r^2 \sin\theta}\boldsymbol{\phi} & \frac{1}{r}\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$