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Electrical Fault Analysis

A Short Course on Short Circuits

Fundamentals / Fault Types / Fault Analysis / MVA Method / Smart Grid / IEEE Standards

by

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Course 543

3 PDH (3 Hours)

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Nomenclature[1]

a	phase	-
a	turns ratio	-
a	ratio of transformation	-
a_n, b_n	Fourier coefficients	-
A	ABCD parameter	-
A	area	m^2
B	ABCD parameter	-
B	magnetic flux density	T
B	magnetic flux density	T
B	susceptance	S, Ω^{-1}, or mho
b	phase	-
c	speed of light	m/s
C	capacitance	F
C	ABCD parameter	-
c	phase	-
CF	crest factor	-
D	ABCD parameter	-
D	distance	m
E	electric field strength	V/m
E	energy	J
E	voltage (generated)	V
E	electromotive force	V
f	form factor	-
f	frequency	Hz, s^{-1}, cycles/s
f_{droop}	frequency droop	Hz/kW
G	conductance	S, Ω^{-1}, or mho
GMD	geometric mean distance	m
GMR	geometric mean radius	m
h	specific enthalpy	kJ/kg
i	variable current	A
I	effective (rms) or DC current	A
I	rms phasor current	A

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related, and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

I^*	complex conjugate of current	A
l	length	m
L	inductance	H
m	mass	kg
M	mutual inductance	H
N	number of turns	-
n_s	synchronous speed	r/min or min^{-1}
p	instantaneous power	W
p	pressure	Pa
P	number of poles	-
P	power	W
pf	power factor	-
pu	per unit	-
Q	reactive power	VAR
Q	heat	J
r	radius	m
R	resistance	Ω
s	specific entropy	$\text{kJ/kg} \cdot \text{K}$
S	apparent power or complex power	VA
SWR	standing wave ratio	-
t	time	s
T	period	s
T	temperature	ºC or K
υ	variable voltage	V
v	velocity (speed)	m/s
V	rms phasor voltage	V
V_{droop}	voltage droop	V/kVAR
VR	voltage regulation	-
W	work	kJ
x	variable	-
X	reactance	Ω
y	admittance per unit length	S/m
Y	admittance	S, Ω^{-1}, or mho
z	impedance per unit length	Ω/m
Z	complex number	-
Z	impedance	Ω
Z_0	characteristic impedance	Ω

Symbols

α	turns ratio	-
α	attenuation constant	Np/m
α	thermal coefficient of resistance	1/°C
β	phase constant	rad/m
γ	propagation constant	rad/m
Γ	reflection coefficient	-
δ	skin depth	m
Δ	change, final minus initial	-
ε	permittivity	F/m
ε_0	free-space permittivity	8.854×10^{-12} F/m
ε_r	relative permittivity	-
η	efficiency	-
θ	phase angle	rad
κ	coupling coefficient	-
μ	permeability	H/m
μ_o	free-space permeability	1.2566×10^{-6} H/m
μ_r	relative permeability	-
ξ	ratio of radii	-
ρ	resistivity	$\Omega \cdot \text{m}$
σ	conductivity	S/m
ϕ	phase angle difference	rad
ϕ	impedance angle or power angle	rad
ϕ	current angle	rad
ϕ	phase angle difference	rad
ϕ_{pf}	power factor angle	rad
ω	armature angular speed	rad/s

Subscripts

ϕ	phase
0	zero sequence
0	characteristic
0	free space (vacuum)
0,o	initial (zero value)
1	positive sequence
1	primary
2	negative sequence
2	secondary
ab	a to b
AC	alternating current
avg, ave	average
bc	b to c
c	controls or critical
c	core
C	capacitive or capacitor
ca	c to a
Cu	copper
d	direct
DC	direct current
e	eddy current
e	equivalent
eff	effective
ext	external
E	generated voltage
f	final / frequency
fl	full load
g or *gen*	generator
h	hysteresis
i	imaginary
int	internal
I	current
l	line
l	per unit length

L	inductor or load
ll	line-to-line
ln	line to neutral
m	motor
m	maximum
m	mutual
max	maximum
n	neutral
nl	no load
O	origin
oc	open circuit
p	peak
p	phase
p	primary
pf	power factor
pri	primary
ps	primary to secondary
pu	per unit
q	quadrature
rms	root mean square
r	real
R	receiving end
R	resistance or resistive
s	secondary, source, synchronous
S	sending end
sc	short circuit
sec	secondary
t	terminal or total
thr	threshold
trans	transmission
V	potential difference between two points
w	wave
Z	impedance

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COURSE INTRODUCTION

The information is primarily from the author's books, Refs. [A] and [B] with the NESC information in Ref. [C]. The coverage of the NESC does not include end-users buildings—this is covered by the NEC, Ref. [D]. Information useful in many aspects of electric engineering may be found in [E] and [F] as well as the appendices. Reference [G] has detailed descriptions of analysis techniques. Reference [H] provides detailed engineering review. Reference [I] provides indepth explanation of the per-unit system often used in such engineering. Reference [J] covers many terms in EE with excellent definitions and explanations.

FUNDAMENTALS

Alternating waveforms have currents and voltages that vary with time in a regular and symmetrical manner. Waveform shapes include square, sawtooth, and triangular, along with many variations on these themes. However, for most applications in electrical engineering the variations are sinusoidal in time. In this book, unless otherwise specified, currents and voltages are sinusoidal.[2]

When sinusoidal, the waveform is nearly always referred to as AC—that is, alternating current—indicating that the current is produced by the application of a sinusoidal voltage. This means that the flow of electrons changes directions, unlike DC circuits, where the flow of electrons is unidirectional (though the magnitude can change in time).

A circuit is said to be in a steady-state condition if the current and voltage time variation is purely constant (DC) or purely sinusoidal (AC).[3] In this course, unless otherwise specified, all circuits are in a steady-state condition.

FAULTS AND FAULT CURRENT

A *fault* is an unwanted connection (i.e., a short circuit) between a line and ground or another line. Although the fault current is usually very high before circuit breakers trip, it is not infinite, because the transformers and transmission line have finite impedance up to the fault point. If the line impedance is known, the fault current can be found by Ohm's law.[4]

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[2] Nearly all periodic functions can be represented as the sum of sinusoidal functions, which simplifies the mathematics needed.
[3] Steady-state AC may have a DC offset.
[4] Equation 61 ignores the transient (DC) current component and the relatively small current flowing in the wire before the fault occurs. Although the fault current has a transient component, it dies out so quickly that it is insignificant.

Equation 1: Fault Current

$$V = I_{\text{fault}} Z$$

FAULT ANALYSIS: SYMMETRICAL

A fault is any defect in a circuit, such as an open circuit, short circuit, or ground. Short-circuit faults, called *shunt faults*, are shown in Fig. 1. Open-circuit faults are called *series faults*. Any fault that connects a circuit to ground is termed a ground fault. The balanced three-phase short circuit shown in Fig. 1(d) is one of the least likely, yet most severe, faults and determines the ratings of the supplying circuit breaker.

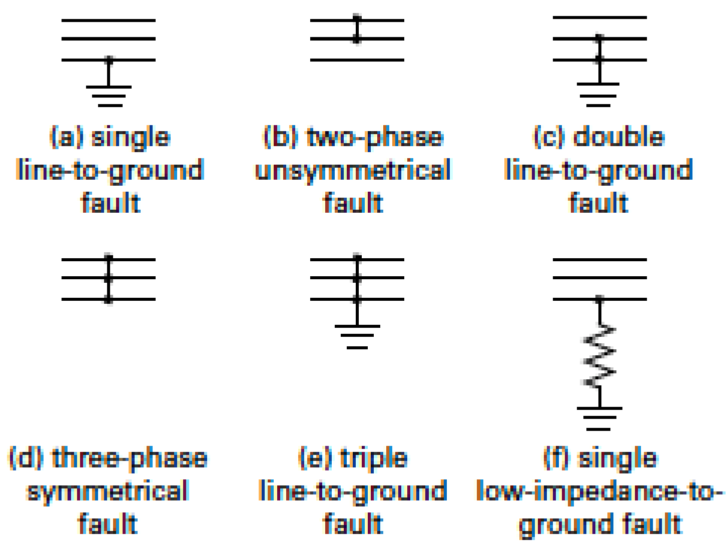

Figure 1: Fault Types, from Most Likely to Lease Likely

A three-phase symmetrical fault, such as that in Fig. 1(d), can be divided into three stages, as shown in Fig. 2. During the subtransient period, which lasts only for a few cycles, the current rapidly decreases and the synchronous reactance, X_s, changes to the subtransient reactance, X_d . There are two reactances: the direct axis reactance, X_d, that lags the generator voltage by 90°, and a quadrature reactance, X_q, that is in phase with the generator voltage.[5] The model for a synchronous generator changes from that in Fig. 3(a) to that in Fig. 3(b). The sudden change in armature current results in a lower armature reactance, but the current through the leakage reactance must remain the same for continuity of energy. For proper modeling, it is necessary to

[5] During faults, the power factor is low and the quadrature reactance can be ignored.

change the voltage, E_g, to E_g''. The voltage E_g'' is calculated for the subtransient interval *just prior to the initiation of the fault* using Eq. 2.[6]

Equation 2: Subtransient Voltage

$$E''_{g,m} = V_t + jI_L X''_d$$

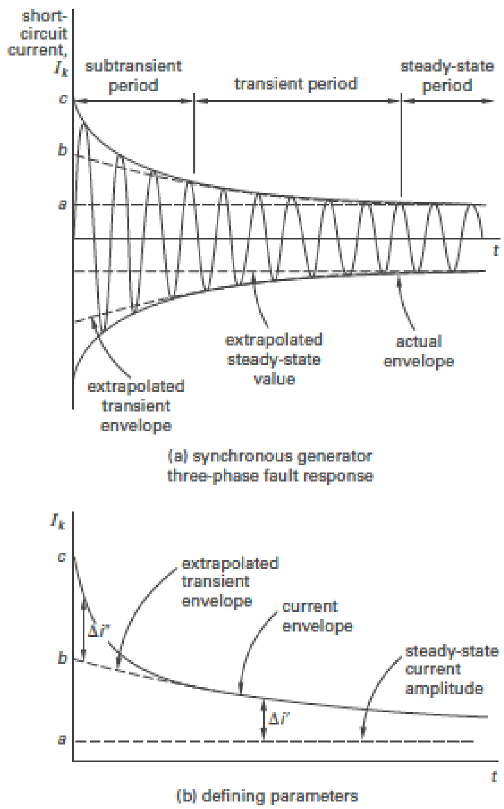

Figure 2: Symmetrical Fault Terminology

[6] The general form for the synchronous generator voltage is $E_g = V_t + jIX$.

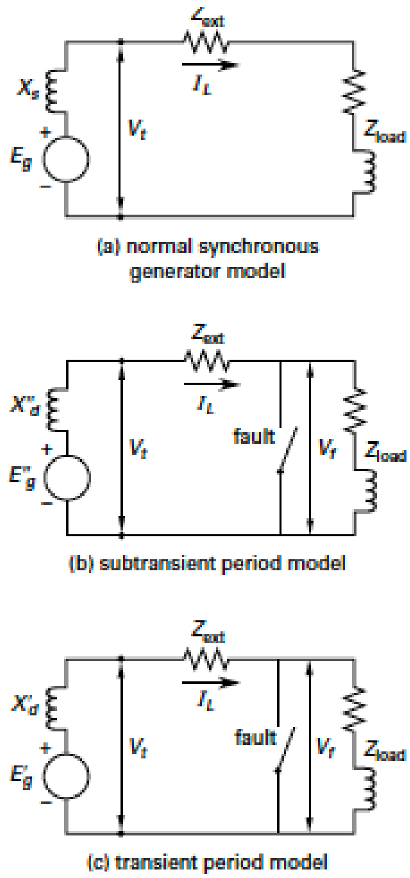

Figure 3: Synchronous Generator Fault Models

During the *transient period*, a similar situation exists and the model for the synchronous generator changes from that in Fig. 3(b) to that in Fig. 3(c). The correct generator voltage for this period. is *calculated just prior to the initiation of the fault* using

Equation 3: Transient Voltage

$$E'_g = V_t + jI_L X'_d$$

Both E''_g and E'_g depend on the impedance of the load and the resulting current prior to fault initiation. The reactances associated with the subtransient, transient, and steady-state periods have increasing values; that is, $X''_d < X'_d < X_d$. Typical reactances of three-phase synchronous machines are given in Table 1. The corresponding short-circuit currents have decreasing magnitudes; that is, $|I''| > |I'| > |I|$. These currents are the symmetrical rms currents associated with a fault.

Table 1: Typical Reactances of Three-Phase Synchronous Machines

(Typical Values given Above Bars / Ranges Below Bars)[7]

	X_d (unsaturated)	X_q (rated current)	X'_d (rated voltage)	X''_d (rated voltage)
two-pole turbine generators	1.20 / 0.95–1.45	1.16 / 0.92–1.42	0.15 / 0.12–0.21	0.09 / 0.07–0.14
four-pole turbine generators	1.20 / 1.00–1.45	1.16 / 0.92–1.42	0.23 / 0.20–0.28	0.14 / 0.12–0.17
salient-pole generators and motors (with dampers)*	1.25 / 0.60–1.50	0.70 / 0.40–0.80	0.30* / 0.20–0.50	0.20* / 0.13–0.32
salient-pole generators (without dampers)	1.25 / 0.60–1.50	0.70 / 0.40–0.80	0.30* / 0.20–0.50	0.30* / 0.20–0.50
capacitors (air-coded)	1.85 / 1.25–2.20	1.15 / 0.95–1.30	0.40 / 0.36–0.50	0.27 / 0.19–0.30
capacitors (hydrogen-coded at ½ psi)	2.20 / 1.50–2.65	1.35 / 1.10–1.55	0.48 / 0.36–0.60	0.32 / 0.23–0.36

*High-speed units tend to have low reactance, and low-speed units tend to have high reactance.

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In Fig. 2, the current at point *c*, I_c, is the *subtransient current,* I'', and is often called the *initial symmetrical rms current*. The *DC component of the fault current* is the exponentially decaying portion—that is, the exponential portion—of the short-circuit current. This DC component and the prospective short-circuit current are shown in Fig. 4. The subscript *k*, indicates short-circuit parameters in many standards.[8]

The initial symmetrical rms current (the subtransient current), I'', should not be confused with the *initial symmetrical short-circuit current*, I_k , shown in Fig. 4 and used in IEEE standards and in various short-circuit analysis software programs. I''_k is the maximum rms value of the prospective (available) short-circuit current with the impedance at the zero-time value (prior to the fault). The subtransient short-circuit current, I'', is the fault current flowing with the reactance equal to the

[7] This data is dated and has likely been refined. Use the most accurate available.

[8] These standards include IEEE 551, *Recommended Practice for Short-Circuit Calculations in Industrial and Commercial Power Systems (IEEE Violet Book*), and IEEE 399, *Recommended Practice for Industrial and Commercial Power Systems Analysis* (*IEEE Brown Book*). These two standards reference international standard IEC 60909-0, *Short-Circuit Currents in Three-Phase A.C. Systems—Part 0*: *Calculation of Currents*, which uses the *k* subscript used in Fig. 3. Slight differences exist between the U.S. standards and the international standard. Both sets of standards, however, are used in short-circuit analysis software, and the terminology used in both may be encountered by an electrical engineer.

subtransient reactance (after fault initiation), X_d''.[9] The subtransient reactance is used in determining the *peak short-circuit current*, i_p, shown in Fig. 4. This peak current is the maximum possible value of the short-circuit current and is equivalent to the subtransient current, I''.

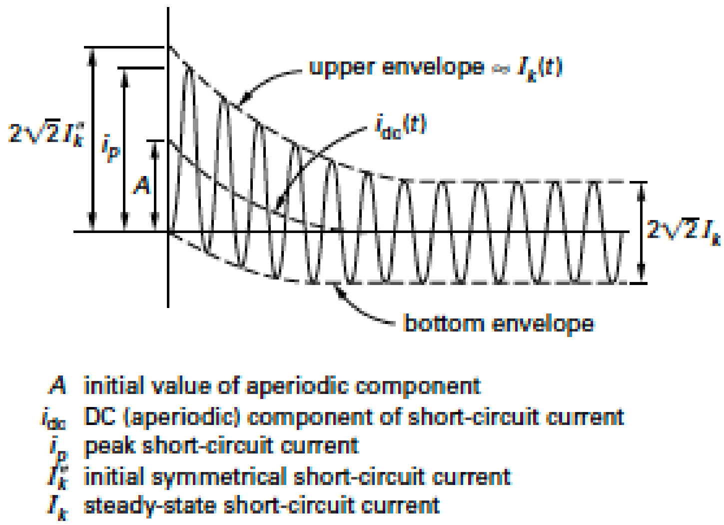


Figure 4: IEEE Nomenclature for Three-Phase, Short-circuit Current

In a synchronous generator, the subtransient time and reactance depend on the damping in the rotor circuit, the transient time and reactance depend on the damping in the excitation circuits, and the steady-state time and reactance depend on the stator circuit.[10] Most calculations use only the direct axes reactances. Ignoring the quadrature reactances results in an error of approximately 10%, and the values are higher when quadrature reactance is taken into account. Using the quadrature values makes the calculations complex and gains little in accuracy.

The subtransient time period lasts a few cycles and is defined in the IEEE standards as the time it takes for the rapidly changing component of the direct-axis, short-circuit current to decrease to 1/e, or 0.368, of its initial value. The transient time period is defined in the standards as the time it takes for the slowly changing component of the direct-axis, short-circuit current to decrease to 1/e, or 0.368, of its initial value.[11] The steady-state period lasts from the end of the transient period until the circuit breaker or other interrupting device acts to remove the fault.

[9] This is consistent with Power System Analysis by Grainger and Stevenson Ref [G].
[10] The steady-state reactance is the normal generator-synchronous reactance, X_d.
[11] Subtransient times vary with system electrical values, but "a few" cycles is often the range: approximately 0 cycles to 4 cycles, or 0 ms to 66 ms. Transient times last approximately 6 cycles, or from 100 ms to as long as 5 s.

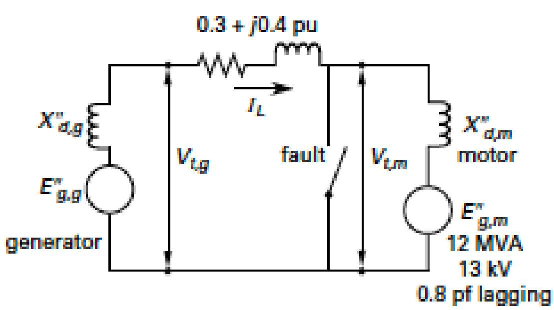
When a synchronous motor is part of a system, it is considered a synchronous generator for fault analysis.[12] Circuit breakers must be able to interrupt fault currents at the rated voltage. The usual fault ratings are in kVA or MVA. The fault rating is the product of the interrupt current capacity and the line-to-line kV rating.

Example 1

A synchronous generator and motor are rated for 15 MVA, 13.9 kV, and 25% subtransient reactance. The line impedance connecting the generator and motor is 0.3 + *j*0.4 pu, given with the ratings as the base. The motor is operating at 12 MVA, 13 kV, with a 0.8 pf lagging. Determine the per-unit equivalent generated voltage for the motor, $E''_{g,m}$, with the expectation of a fault at the motor terminals.

Solution

The new one-line diagram illustrating the situation is



Start by determining the base quantities for this three-phase system.

[12] The motor actually becomes a generator when a fault occurs. The spinning motor changes the mechanical energy it possesses into electrical energy, supplying the fault and slowing in the process.

$$S_{\text{base}} = S_t = 15 \text{ MVA} \quad (15\times10^6 \text{ VA})$$

$$V_{\text{base}} = V_l = 13.9 \text{ kV} \quad (13.9\times10^3 \text{ V})$$

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{15\times10^6 \text{ VA}}{\sqrt{3}(13.9\times10^3 \text{ V})} = 623 \text{ A}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{\sqrt{3}\, I_{\text{base}}} = \frac{13.9\times10^3 \text{ V}}{\sqrt{3}\left(623 \text{ A}\right)} = 12.9\ \Omega$$

Now, determine the per-unit quantities of interest, that is, the voltage at the fault site (the terminals of the motor), and the current flowing.

$$V_{pu} = \frac{V_{\text{actual}}}{V_{\text{base}}} = \frac{13\times10^3 \text{ V}}{13.9\times10^3 \text{ V}} = 0.935 \text{ pu}$$

$$\text{I}_{pu} = \frac{I_{\text{actual}}}{I_{\text{base}}} = \frac{\dfrac{S_m}{\sqrt{3}\left(I_m\right)}}{I_{\text{base}}} = \frac{\dfrac{12\times10^6 \text{ VA}}{\sqrt{3}\left(13\times10^3 \text{ V}\right)}}{0.623\times10^3 \text{ A}} = 0.860 \text{ pu}$$

The power factor is 0.8 lagging, so

$$\text{lagging pf} = \cos\theta = 0.8$$
$$\theta = \arccos 0.8 = -36.9^\circ$$

The line current, I_l, is

$$I_l = I_{pu}\angle\theta = 0.860 \text{ pu}\angle -36.9^\circ$$
$$= 0.688 - j0.516 \text{ pu}$$

The equivalent generated voltage for the motor can now be found. Writing the equation for the voltage from the motor terminals to the motor armature gives

$$
\begin{aligned}
E''_{g,m} &= V_{t,m} - jX''_d I_l \\
&= 0.935 \text{ pu} - j(0.25)(0.688 - j0.516 \text{ pu}) \\
&= 0.806 - j0.172 \text{ pu} \\
&= 0.824 \text{ pu}\angle -12^\circ
\end{aligned}
$$

Example 2

Determine the per-unit equivalent generated voltage for the generator, $\boldsymbol{E''_{g,g}}$, for the system in Example 1.

Solution

The motor terminal voltage is known and the line current is known. An equation for the equivalent generated voltage, written from the known quantities at the motor terminals, is

$$\boldsymbol{E''_{g,g} = V_{t,m} + I_l Z}$$

The impedance is from the motor terminals back to the generator, which is the only unknown. From Ex. 1, since there is a 25% subtransient reactance, $Z_g = j0.25$ pu.

$$
\begin{aligned}
Z = Z_l + Z_g &= (0.3 + j0.4 \text{ pu}) + j0.25 \text{ pu} \\
&= 0.3 + j0.65 \text{ pu} \\
&= 0.715 \text{ pu}\angle 65^\circ
\end{aligned}
$$

Substituting gives

$$
\begin{aligned}
E''_{g,g} &= V_{t,m} + I_l Z \\
&= 0.935 \text{ pu} + (0.715 \text{ pu}\angle 65^\circ)(0.860 \text{ pu}\angle -36.9^\circ) \\
&= 1.5 \text{ pu}\angle 11.1^\circ
\end{aligned}
$$

Example 3

Determine the per-unit subtransient period current for the motor in Ex. 1.

Solution

A fault at the motor terminals results in a current given by

$$E''_{g,m} = I_m + jX''_d$$

$$I_m = \frac{E''_{g,m}}{jX''_d} = \frac{0.824 \text{ pu}\angle -12^\circ}{0.25 \text{ pu}\angle 90^\circ} = 3.3 \text{ pu}\angle -102^\circ$$

Example 4

Determine the per-unit subtransient period current for the generator in Ex. 1.

Solution

The generator current that flows to the fault is given by

$$E''_{g,g} = I_g Z$$

$$I_g = \frac{E''_{g,g}}{Z} = \frac{1.5 \text{ pu}\angle 11.5^\circ}{0.715 \text{ pu}\angle 65^\circ} = 2.09 \text{ pu}\angle -53.9^\circ$$

Example 5

What is the actual fault current for the circuit in Ex. 1?

Solution

The actual fault current is the total current through the fault from the generator and the motor. The synchronous motor acts as a generator to the fault as it gradually loses speed. The subtransient period is too short for the motor to lose a significant amount of speed, so it can be treated as a generator. The total per-unit current is

$$\begin{aligned} I_{pu} &= I_m + I_g = 3.3 \text{ pu}\angle -102^\circ + 2.09 \text{ pu}\angle -53.9^\circ \\ &= 4.95 \text{ pu}\angle -83.7^\circ \end{aligned}$$

The actual current is[13]

$$\begin{aligned} I_{\text{actual}} &= I_{pu} I_{\text{base}} = \left(4.95 \text{ pu}\angle -83.7^\circ\right)\left(0.62 \text{ kA}\right) \\ &= 3.07 \text{ kA}\angle -83.7^\circ \end{aligned}$$

Example 6

Determine the minimum rating necessary for the motor and generator circuit breakers to protect against the fault described in Ex. 1.

Solution

The minimum rating is the base apparent power, S_{base}, multiplied by a factor representing the per-unit current flowing above the base current, $I_{pu,\text{fault}}$. For the motor circuit breaker,

$$\begin{aligned} S_{\text{rating}} &= I_{pu,\text{fault}} S_{\text{base}} = (3.3)(15 \text{ MVA}) \\ &= 49.5 \text{ MVA} \end{aligned}$$

For the generator circuit breaker,

$$\begin{aligned} S_{\text{rating}} &= I_{pu,\text{fault}} S_{\text{base}} = (2.09)(15 \text{ MVA}) \\ &= 31.4 \text{ MVA} \end{aligned}$$

FAULT ANALYSIS: UNSYMMETRICAL

Any fault that is not a three-phase short is an unsymmetrical fault, also called an *asymmetrical fault*. Examples include the line-to-line and line-to-ground faults shown in Fig. 7(a-c) and Fig. 7(e

[13] It should be mentioned that what is showing is a common use of units kA, kV, and kVA. Though, one can/should avoid these and instead use A, V, and VA and a power of 10 to minimize mathematical errors.

and f). Such faults are more common than three-phase shorts, but they are more difficult to analyze because they result in uneven phase voltages and currents. Nevertheless, unsymmetrical faults can be analyzed by separating unbalanced phasor components into three sets of symmetrical components as shown in Fig. 5.

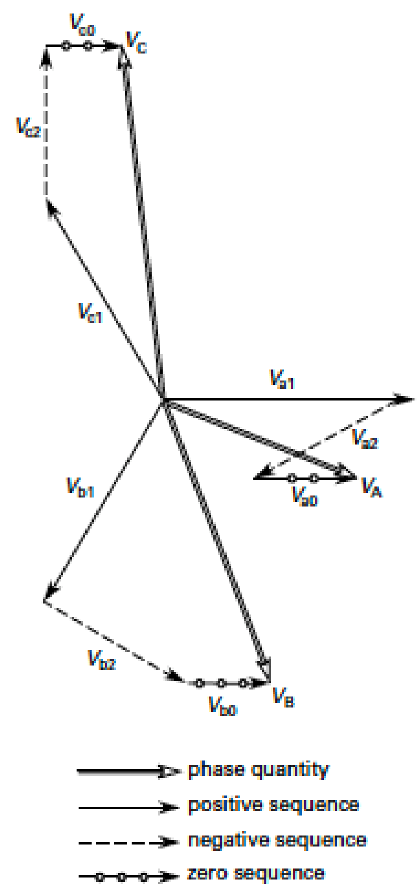


Figure 5: Phasor Diagram—Symmetrical Components of Unbalanced Phasors

The three sets of symmetrical components shown in Fig. 5 are referred to as the *positive-sequence*, *negative-sequence*, and *zero-sequence* components of the unsymmetrical phasors. The positive-sequence phasors, shown in Fig. 6(a), are three equal magnitude phasors rotating counterclockwise in the sequence a, b, c. Positive-sequence phasors are 120 electrical degrees apart and sum to zero. Positive sequence components are the ones normally used in electrical engineering. Such components represent balanced three-phase generators, motors, and transformers. The subscript 1 is normally used to indicate the positive sequence.

The negative-sequence phasors, shown in Fig. 6(b), are three equal-magnitude phasors rotating counterclockwise in the sequence a, c, b. They can also be represented as a mirror image of the positive-sequence phasors rotating in the clockwise direction. Negative sequence phasors are 120 electrical degrees apart and sum to zero. The negative-sequence reactance is the average of the subtransient direct and quadrature reactances, as given in Eq. 4. The subscript 2 is normally used to indicate the negative sequence.

Equation 4: Negative Sequence Reactance

$$X_2 = \tfrac{1}{2}\left(X''_d + X''_q\right)$$

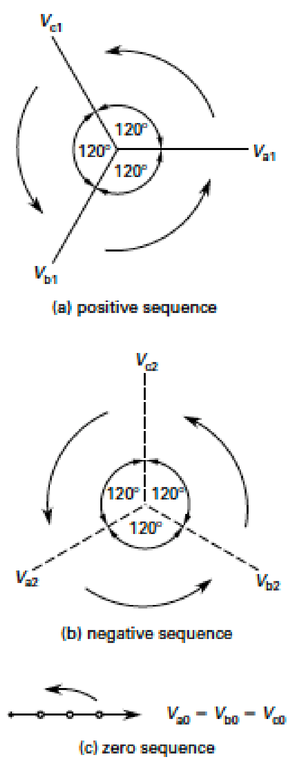


Figure 6: Components of Unsymmetrical Phasors

The zero-sequence phasors, shown again in Fig. 6(c), are three equal-magnitude phasors coincident in phase sequence and rotating counterclockwise. The subscript 0 is normally used to indicate the zero sequence.

The unsymmetrical phasors of Fig. 5 are represented *in terms of their symmetrical components* by Eq. 5 through Eq. 7.

Equation 5: Unsymmetrical Phasor A

$$V_{\mathrm{A}} = V_{a0} + V_{a1} + V_{a2}$$

Equation 6: Unsymmetrical Phasor B

$$V_{\mathrm{B}} = V_{b0} + V_{b1} + V_{b2}$$

Equation 7: Unsymmetrical Phasor C

$$V_{\mathrm{C}} = V_{c0} + V_{c1} + V_{c2}$$

Consider an operator *a*, defined as 1∠120°, a unit vector with an angle of 120°.[14] The properties of *a* are as follows.

Equation 8: Properties of Operator *a*

$$a = 1\angle 120^\circ = 1 \times e^{j120^\circ} = -0.5 + j0.866$$

Equation 9: Properties of Operator *a*

$$a^2 = 1\angle 240^\circ = 1 \times e^{j240^\circ} = -0.5 - j0.866 = a^*$$

Equation 10 Properties of Operator *a*

$$a^3 = 1\angle 360^\circ = 1\angle 0^\circ$$

Equation 11: Properties of Operator *a*

$$a^4 = a$$

Equation 12: Properties of Operator *a*

$$a^5 = a^2$$

Equation 13: Properties of Operator *a*

$$a^6 = a^3$$

Equation 14: Properties of Operator *a*

$$1 + a + a^2 = 0$$

Using Eq. 5 through Eq. 7 and the concept of the phasor **a**, the unsymmetrical components can be *represented in terms of a single phase*. For example, using phase A gives

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[14] Although *a* is a vector, it is not generally written as **a**. In this it resembles the operator j, which is equal to 1∠90° but is rarely written as **j**.

Equation 15: Unsymmetrical Phasor A in Terms of Phase A

$$V_{\mathrm{A}} = V_{a0} + V_{a1} + V_{a2}$$

Equation 16: Unsymmetrical Phasor B in Terms of Phase A

$$V_{\mathrm{B}} = V_{a0} + a^2 V_{a1} + a V_{a2}$$

Equation 17: Unsymmetrical Phasor C in Terms of Phase A

$$V_{\mathrm{C}} = V_{a0} + a V_{a1} + a^2 V_{a2}$$

These individual equation in matrix form, which is useful for computer analysis of same, follows.

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix}$$

Solving for the sequence components from Eq. 15 through Eq. 17 gives the following.

Equation 18: Zero Sequence Components

$$V_{a0} = \tfrac{1}{3}\left(V_{\mathrm{A}} + V_{\mathrm{B}} + V_{\mathrm{C}}\right)$$

Equation 19: Positive Sequence Components

$$V_{a1} = \tfrac{1}{3}\left(V_{\mathrm{A}} + a V_{\mathrm{B}} + a^2 V_{\mathrm{C}}\right)$$

Equation 20: Negative Sequence Components

$$V_{a2} = \tfrac{1}{3}\left(V_{\mathrm{A}} + a^2 V_{\mathrm{B}} + a V_{\mathrm{C}}\right)$$

These individual equations can be represented in matrix form as follows.

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}$$

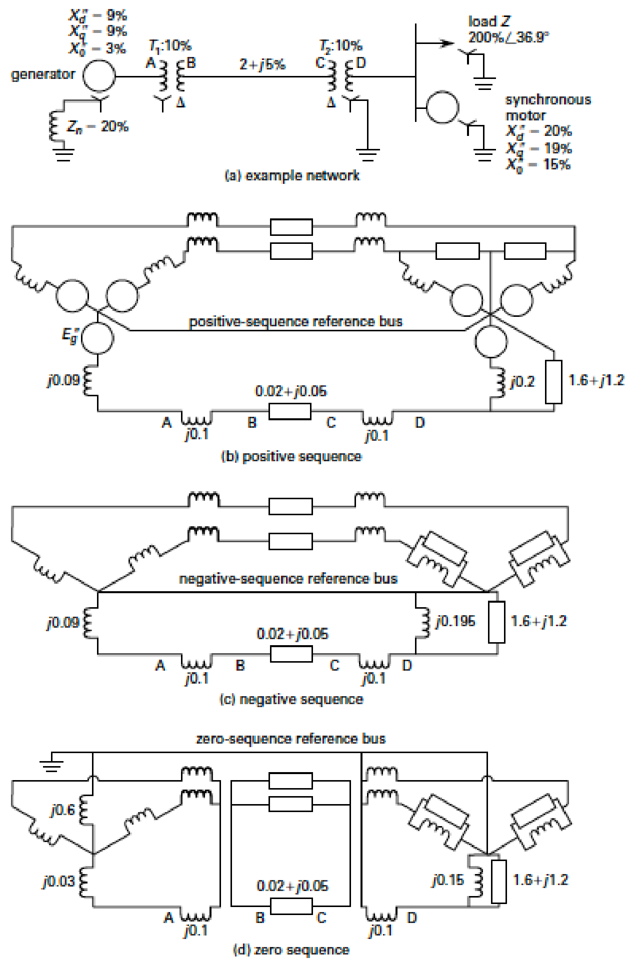

Figure 7: Sample Sequence Networks

Equations similar to Eq. 15 through Eq. 17 exist for currents as well. Sequence impedances may also be defined. Impedance through which only positive-sequence currents flow is called a positive-sequence impedance. Positive-sequence networks are normally used in electrical engineering, and an example is given in Fig. 7(b). Negative-sequence impedance is similar to positive-sequence impedance with the reactance given by Eq. 4. In addition, a negative-sequence network omits all positive-sequence generators. A sample negative-sequence network is shown in Fig. 7(c).

Zero-sequence impedance is significantly different from positive-sequence impedance. The only machine impedance seen by the zero-sequence impedance is the leakage reactance, X_0. Series reactance is greater than positive-sequence reactance by a factor of 2 to 3.5. A sample zero-sequence network is shown in Fig. 7(d). Only a wye-connected load with a grounded neutral permits zero-sequence currents. Only a delta-connected transformer secondary permits zero-sequence currents. Figure 8 shows zero-sequence impedances for various configurations. The use of sequence networks simplifies fault calculations.

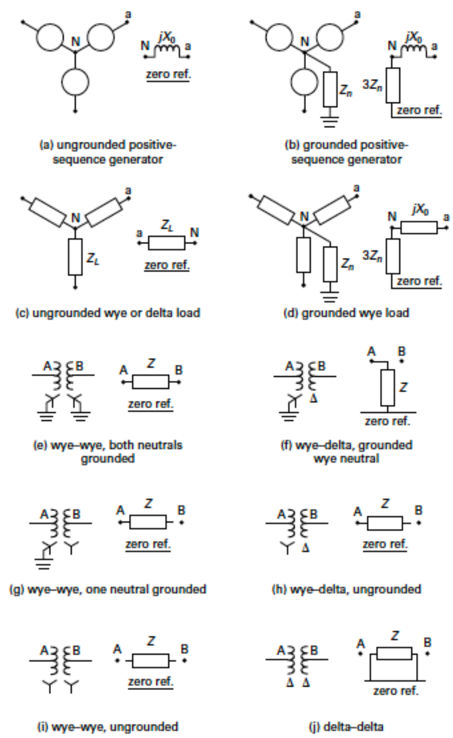

Figure 8: Zero-Sequence Impedances

FAULT ANALYSIS: THE MVA METHOD

The fault analysis techniques in the earlier sections provide detailed information on the fault, although they also require significant calculation time. Information on those techniques may be found in IEEE Standard 141 titled *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants*. International Standard IEC 60909 also provides guidance for short-circuit analysis that differs from the IEEE guidance and, in some ways, is more detailed. Numerous methods exist, including *ohmic*, *Thevenin*, *E/Z*, and *E/X* methods. Many of these short-circuit analysis methods require the use of computers.

Importantly, a simplified method, which nevertheless captures the worst-case fault current, is called the *MVA method*. The MVA method doesn't need significant calculation time or software support. This method calculates the fault current that will flow in a system or component for *unit voltage supplied from an infinite source*. (The infinite source assumption means that the power sources are treated as reactors, that is, inductive components with no resistance, and the voltage is steady at the base value throughout.) The MVA method uses the per-unit system, maintains the voltage at a 1 pu value, and calculates the fault (or short-circuit) apparent power, S_{fault} or S_{sc}, and the fault (or short-circuit) current, I_{fault} or I_{sc}. Equation 21 and Eq. 22 are used to calculate the maximum fault power and fault current.

Equation 21: MVA Method, Short-Circuit Apparent Power

$$S_{sc} = \frac{S_{\text{base}}}{Z_{pu}}$$

Equation 22: MVA Method, Short-Circuit Current

$$I_{sc} = \frac{S_{sc}}{\sqrt{3}V_{\text{base}}}$$

Variations of this method are used, including one that adds the power sources (i.e., the MVA sources) as vectors. Series power sources are treated as capacitors in series, given that the power transferred to the fault must go through both sources. For example, a generator has to pass its power through an intervening transformer. Parallel power sources are treated as capacitors in parallel, given that power from both is available to the fault.[15]

Motors are treated as generators during short circuit fault analysis because rotational energy is converted to electrical energy and transferred to the fault during such events. Standard practices vary as to the contributions from various size motors.

Finally, the equations used in this course assume symmetrical faults. The power may be calculated based on the positive-, negative-, and zero-sequence impedances, and power may then be combined to provide results for unsymmetrical faults.

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[15] This paragraph and the one that immediately follows encapsulate the MVA method and are worth study. The two examples that follow will attempt to display the usefulness and ease of calculation for this method.

Example 7

Derive the equation for fault apparent power, S_{sc}, that is Eq. 21.

Solution

The base apparent power is

$$S_{\text{base}} = \sqrt{3} I_{\text{base}} V_{\text{base}} \quad \text{[I]}$$

Rearranging for the base current gives

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3} V_{\text{base}}} \quad \text{[II]}$$

The fault apparent power and current will use the base voltage, as this provides the worst-case condition and allows for per-unit calculations across transformers. Changing Eq. I and Eq. II to their fault versions gives

$$S_{sc} = \sqrt{3} I_{sc} V_{\text{base}} \quad \text{[III]}$$

$$I_{sc} = \frac{S_{sc}}{\sqrt{3} V_{\text{base}}} \quad \text{[IV]}$$

Equation IV is in the form of Eq. 22. The short-circuit (fault) current, I_{sc}, is the actual current that flows in the fault. Using the per-unit system,

$$I_{sc} = I_{\text{actual}} = I_{\text{base}} I_{pu} \quad \text{[V]}$$

The per-unit current is given by

$$I_{pu} = \frac{V_{pu}}{Z_{pu}} \quad \text{[VI]}$$

The per-unit value of the voltage is 1 pu, given that the base voltage is being used. Substituting the number 1 into Eq. VI yields Eq. VII.[16]

$$I_{pu} = \frac{1}{Z_{pu}} \quad \text{[VII]}$$

Substituting Eq. VII into Eq. V gives

$$I_{sc} = I_{\text{actual}} = I_{\text{base}}\left(\frac{1}{Z_{pu}}\right) \quad \text{[VIII]}$$

Substituting Eq. VIII into Eq. III gives

$$S_{sc} = \sqrt{3}\left(I_{\text{base}}\left(\frac{1}{Z_{pu}}\right)\right)V_{\text{base}} = \frac{\sqrt{3}I_{\text{base}}V_{\text{base}}}{Z_{pu}} \quad \text{[IX]}$$

Equation IX is used to show the base apparent power in Eq. X that follows.

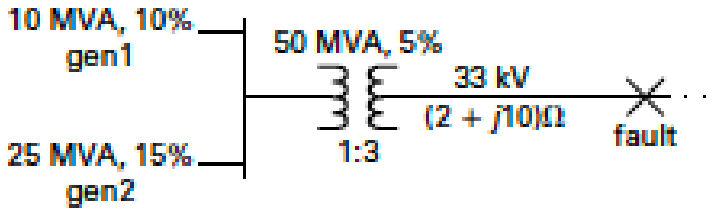
$$S_{sc} = \frac{\sqrt{3}I_{\text{base}}V_{\text{base}}}{Z_{pu}} = \frac{S_{\text{base}}}{Z_{pu}} \quad \text{[X]}$$

Equation 10 is Eq. 21, and it completes the derivation.[17]

Example 8

Consider the one-line diagram for a three-phase generator reactor system in the illustration. The power ratings and per-unit impedances based on those ratings are given. Assume a worse-case symmetrical fault. Using the MVA method, what is the maximum short-circuit current at the fault location shown?

[16] Equation VIII along with Eq. 2 provide a quick method of calculating the first-order short-circuit current that is very useful in the field.
[17] Or, to use a term learned in college that now presents itself as an opportunity to utilize, QED, or quod erat demonstrandum meaning "which was to be demonstrated".



Solution[18]

Using an arbitrary base of 100 MVA for S_{base} and 33 kV for V_{base}, the equation below converts to the following reactance values from the given rating base values to the new base.[19] (The reactances, or reactors, are the source of energy to the fault.) A base voltage of 33 kV is used to ensure the proper per-unit value was calculated for the line impedance because *the conversion of actual to per-unit values must use the voltage present in the given section of the distribution system*. Additionally, the fault is in the 33 kV section of the distribution system, so using 33 kV ensures the correct current calculation for the fault current. Reactance values, *X*, are used instead of impedance values, *Z,* because the sources are considered infinite, that is, no resistance is present.

$$X_{\text{pu,new}} = X_{\text{pu,old}} \left(\frac{X_{\text{base,old}}}{X_{\text{base,new}}} \right)$$

Because no resistance is present, *X* is equivalent to *Z*. Additionally, the base voltage is unchanged. The result is Eq. [I]

$$Z_{\text{pu,new}} = Z_{\text{pu,old}} \left(\frac{V_{\text{base,old}}}{V_{\text{base,new}}} \right)^2 \left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}} \right) = Z_{\text{pu,old}} \left(\frac{33\ \text{kV}}{33\ \text{kV}} \right)^2 \left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}} \right) = Z_{\text{pu,old}} \left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}} \right) \quad \text{[I]}$$

Now, obtain the per-unit values of the sources of energy for the fault.

$$\mathbf{Z}_{\text{pu,gen1}} = jX_{\text{gen1}} = j0.10 \left(\frac{100\ \text{MVA}}{10\ \text{MVA}} \right) = j1.0\ \text{pu} \quad \text{[II]}$$

[18] This problem will be shown in detail as the author has used it to determine the adequacy of protection against short circuits and to verify computer outputs of same.
[19] This equation was explained in an earlier course, but is used often in such analysis so is presented here.

$$\mathbf{Z}_{\text{pu,gen2}} = jX_{\text{gen2}} = j0.15\left(\frac{100\ \text{MVA}}{25\ \text{MVA}}\right) = j0.6\ \text{pu} \quad \text{[III]}$$

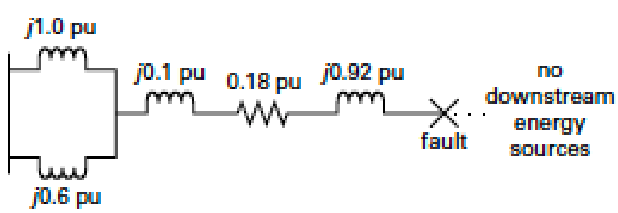
$$\mathbf{Z}_{\text{pu,transformer}} = jX_{\text{transformer}} = j0.05\left(\frac{100\ \text{MVA}}{50\ \text{MVA}}\right) = j0.1\ \text{pu} \quad \text{[IV]}$$

In some texts, only the reactance, *X*, is shown. Even so, when used in per-unit calculations, the *j* is added as a reminder that the value is for a reactive component, and must be added as a vector to any resistive components. Additionally, the impedance, **Z**, may not be shown in the bold format, even though it carries angular information. The form used in Eq. I through Eq. IV is in keeping with IEEE standards.

The impedance of the line is calculated and a rearranged three-phase version is shown.

$$\mathbf{Z}_{pu} = \frac{\mathbf{Z}_{\text{actual}}}{\mathbf{Z}_{\text{base}}} = \frac{\mathbf{Z}_{\text{actual}}}{\left(\dfrac{V_{\text{base}}^2}{S_{\text{base}}}\right)} = \mathbf{Z}_{\text{actual}}\left(\frac{S_{\text{base}}}{V_{\text{base}}^2}\right) = \left(2\ \Omega + j10\ \Omega\right)\left(\frac{100\times10^6\ \text{VA}}{\left(33\times10^3\ \text{V}\right)^2}\right) = 0.18\ \text{pu} + j0.92\ \text{pu} \quad \text{[V]}$$

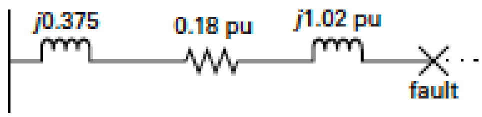
The per-unit values of reactance and impedance do not change as one considers different portions of the distribution system (i.e., each side of the transformer). However, the base voltages do change. The correct base voltage must be used in calculating the per-unit values. *Once complete, the per-unit values may be combined in any way desired*, as shown in the next several steps. The results of Eq. I through Eq. V are as shown in the following illustration.



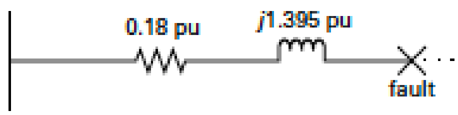
Combining thee generator reactances gives the following.

$$Z_{\text{total,gen}} = jX_{\text{total,gen}} = \frac{\left(jX_{\text{gen1}}\right)\left(jX_{\text{gen2}}\right)}{jX_{\text{gen1}} + jX_{\text{gen2}}} = \frac{\left(j1.0\ \text{pu}\right)\left(j0.6\ \text{pu}\right)}{j1.0\ \text{pu} + j0.6\ \text{pu}} = j0.375\ \text{pu} \quad \text{[VI]}$$

Showing the results of Eq. VI and combining the transformer and line impedances gives



Combining the remaining reactances gives



The total impedance is as follows.

$$\mathbf{Z}_{\text{total}} = (0.18 + j1.395)\ \text{pu} = 1.41\ \text{pu}\angle 83^\circ$$

The angle does not determine the maximum short-circuit current, but it is required for phase calculations and is shown here as clarification only. All the per-unit quantities carry an angle that may be determined if necessary.

Using Eq. 21 and the result of Eq. VII to calculate the short-circuit power.

$$S_{sc} = \frac{S_{\text{base}}}{Z_{PU}} = \frac{100\ \text{MVA}}{1.41\ \text{pu}} = 70.9\ \text{MVA} \quad \left(70.9\times 10^6\ \text{VA}\right) \quad \text{[VIII]}$$

Substitute the result of Eq. VIII into Eq. 22 to determine the fault current at the indicated location.

$$I_{sc} = \frac{S_{sc}}{\sqrt{3}V_{\text{base}}} = \frac{70.9\times 10^6\ \text{VA}}{\sqrt{3}\left(33\times 10^3\ \text{V}\right)} = 1240.4\ \text{A} \quad \text{[IX]}$$

Calculations such as those presented are vital to ensuring a reliable distribution system and to the improvements of the same. To that end, a quick review of grid developments and standards is warranted.

SMART GRID

Though there is no single definition of a *smart grid*, the term generally applies to broad-based electrical distribution systems using some form of computer-based remote control and automation. The "grid" connects power generation to consumers using substations, transformers, switches, wires, communication infrastructure, and other elements. The "smart" components monitor elements of the system, predict electrical demand, and respond to load deviations and transients to provide continuous power.

The properties of smart grids are guided by Title XIII, "Smart Grid," of the Energy Independence and Security Act of 2007 (EISA), which classifies the following types of projects as smart grid projects.

- projects that optimize the monitoring and control of transmission and distribution, including the use of sensors, communication links, and computer software and systems
- communication infrastructure projects to support the grid
- projects incorporating renewables
- microgrid projects supporting highly reliable and resilient *islanded operation* (i.e., smaller grids operating independently of the main grid or through a single interface with that grid)
- automation projects designed to increase information technology, communications, and overall cyber security

IEEE 3000 STANDARDS COLLECTION

The IEEE has established standards to guide the production, distribution, and utilization of electric energy. These IEEE standards are collectively known as the *IEEE 3000 Standards Collection for Industrial and Commercial Power Systems*, formerly as the *IEEE Color Books*. Each book focuses on one aspect of electric power and provides a basis for assessing the requirements for a given electrical project.

- IEEE Standard 141, *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants* (*IEEE Red Book*).
- IEEE Standard 142, *IEEE Recommended Practice for Grounding of Industrial and Commercial Power Systems* (*IEEE Green Book*).

- IEEE Standard 241, *IEEE Recommended Practice for Electric Power Systems in Commercial Buildings* (*IEEE Gray Book*).
- IEEE Standard 242, *IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems* (*IEEE Buff Book*).
- IEEE Standard 399, *IEEE Recommended Practice for Industrial and Commercial Power Systems Analysis* (*IEEE Brown Book*).
- IEEE Standard 446, *IEEE Recommended Practice for Emergency and Standby Power Systems for Industrial and Commercial Applications* (*IEEE Orange Book*).
- IEEE Standard 493, *IEEE Recommended Practice for the Design of Reliable Industrial and Commercial Power Systems* (*IEEE Gold Book*).
- IEEE Standard 551, *Recommended Practice for Calculating AC Short-Circuit Currents in Industrial and Commercial Power Systems* (*IEEE Violet Book*).
- IEEE Standard 602, *IEEE Recommended Practice for Electric Systems in Health Care Facilities* (*IEEE White Book*).
- IEEE Standard 739, *IEEE Recommended Practice for Energy Management in Industrial and Commercial Facilities* (*IEEE Bronze Book*).
- IEEE Standard 902, *IEEE Guide for Maintenance, Operation, and Safety of Industrial and Commercial Power Systems* (*IEEE Yellow Book*).
- IEEE Standard 1015, *IEEE Recommended Practice for Applying Low-Voltage Circuit Breakers Used in Industrial and Commercial Power Systems* (*IEEE Blue Book*).
- IEEE Standard 1100, *IEEE Recommended Practice for Powering and Grounding Electronic Equipment* (*IEEE Emerald Book*).

IEEE RED BOOK

IEEE Standard 141*, IEEE Recommended Practice for Electric Power Distribution for Industrial Plants* (*IEEE Red Book*) compiles the best practices for the design of electric systems for industrial plants, facilities, and associated buildings. It contains information often extracted from other codes, standards, and other technical literature. Chapters cover planning, voltage considerations, *short-circuit calculations*, protective devices, surge protection, power factors, harmonics, power switching, instrumentation, cable systems, busways, energy management, interface considerations, cost, and power system device numbering.

IEEE GRAY BOOK

IEEE Standard 241, *IEEE Recommended Practice for Electric Power Systems in Commercial Buildings* (*IEEE Gray Book*) describes the best practices for the electrical design of commercial buildings. It contains information often extracted from other codes, standards, and other technical literature on the requirements of such structures. Chapters cover load characteristics, voltage considerations, power sources, distribution apparatuses, controllers, services, and equipment vaults. The *IEEE Gray Book* also contains information on electrical equipment rooms, wiring, protection, lighting, space conditioning, automation, expansion, modernization, special occupancy requirements, and energy management.

REFERENCES

A. Camara, John A. *Electrical Engineering Reference Manual.* Belmont, CA: PPI, 2009.
B. Camara, John A. *PE Power Reference Manual*. Belmont, CA: PPI (Kaplan), 2021.
C. Marne, David J., and John A. Palmer. *National Electrical Safety Code*® (NESC®) *2023 Handbook*. New York: McGraw Hill, 2023.
D. Earley, Mark, ed. *NFPA 70, National Electrical Code Handbook*. Quincy, Massachusetts: NFPA, 2020.
E. IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.
F. IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE.
G. Grainger, John J., and William Stevenson, Jr. *Power System Analysis*. New York, McGraw Hill, 1994.
H. Grigsby, Leonard L., ed. *Electric Power Generation, Transmission, and Distribution.* Boca, Raton, FL: CRC Press, 2012
I. Wikipedia, The Free Encyclopedia (2024). "Per-Unit System". https://en.wikipedia.org/wiki/Per-unit_system (accessed FEB 2024).
J. Parker, Sybil P., editor in chief. *McGraw-Hill Dictionary of Scientific and Technical Terms,* 5th ed. New York, McGraw-Hill, 1994.

Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$\text{radians}/(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	$\propto$ or $m_{\propto}$ $\frac{1}{12} m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^{5} Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		–459.67°F (0°R)	–273.16°C (0 K)
Velocity[3]			
Earth escape		3.67×10^{4} ft/sec	1.12×10^{4} m/s
light (vacuum)	c, c_0	9.84×10^{8} ft/sec	2.9979 (3.00) $\times 10^{8}$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.

Appendix C: Fundamental Constants

Table Note 1

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	αB		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/□R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	αN		5.050×10^{-27} J/T
permeability of a vacuum	α0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π	□		1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm-□R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02□F, 0.0888 psia	0.01109□C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol-□R 1.986 BTU/lbmol-□R	8314 J/kmol·K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

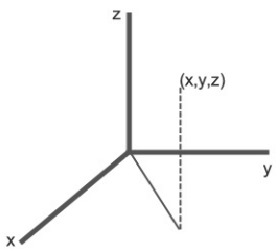
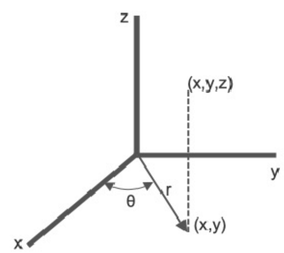
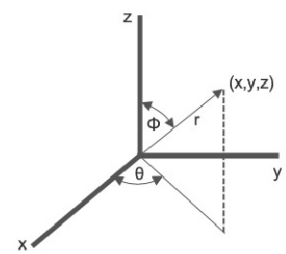
Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Appendix F: Coordinate Systems & Related Operations

Mathematical Operations	**Rectangular Coordinates**	**Cylindrical Coordinates**	**Spherical Coordinates**
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\theta + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\phi + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\theta$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \theta & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\phi & \frac{1}{r}\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

