AC Electrical 101+

Part II

Concepts & Three-Phase Electricity

Fundamentals / Three-Phase Power / Notations / Generation / Distribution Systems / Wye & Delta Connections / Per-Unit Calculations / Unbalanced Loads / Faults

by

John A Camara, BS, MS, PE, TF

Course 540
4 PDH (4 Hours)

PO Box 449
Pewaukee, WI 53072
(888) 564 - 9098
support@pdhacademy.com

Nomenclature[1]

a	phase	-
a	turns ratio	-
a	ratio of transformation	-
a_n, b_n	Fourier coefficients	-
A	ABCD parameter	-
A	area	m^2
B	ABCD parameter	-
B	magnetic flux density	T
B	magnetic flux density	T
B	susceptance	S, Ω^{-1}, or mho
b	phase	-
c	speed of light	m/s
C	capacitance	F
C	ABCD parameter	-
c	phase	-
CF	crest factor	-
D	ABCD parameter	-
D	distance	m
E	electric field strength	V/m
E	energy	J
E	voltage (generated)	V
E	electromotive force	V
f	form factor	-
f	frequency	Hz, s^{-1}, cycles/s
f_{droop}	frequency droop	Hz/kW
G	conductance	S, Ω^{-1}, or mho
GMD	geometric mean distance	m
GMR	geometric mean radius	m
h	specific enthalpy	kJ/kg
i	variable current	A
I	effective (rms) or DC current	A
I	rms phasor current	A

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related, and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

I^*	complex conjugate of current	A
l	length	m
L	inductance	H
m	mass	kg
M	mutual inductance	H
N	number of turns	-
n_s	synchronous speed	r/min or min^{-1}
p	instantaneous power	W
p	pressure	Pa
P	number of poles	-
P	power	W
pf	power factor	-
pu	per unit	-
Q	reactive power	VAR
Q	heat	J
r	radius	m
R	resistance	Ω
s	specific entropy	$\text{kJ/kg}\cdot\text{K}$
S	apparent power or complex power	VA
SWR	standing wave ratio	-
t	time	s
T	period	s
T	temperature	ºC or K
υ	variable voltage	V
v	velocity (speed)	m/s
V	rms phasor voltage	V
V_{droop}	voltage droop	V/kVAR
VR	voltage regulation	-
W	work	kJ
x	variable	-
X	reactance	Ω
y	admittance per unit length	S/m
Y	admittance	S, Ω^{-1}, or mho
z	impedance per unit length	Ω/m
Z	complex number	-
Z	impedance	Ω
Z_0	characteristic impedance	Ω

Symbols

α	turns ratio	-
α	attenuation constant	Np/m
α	thermal coefficient of resistance	1/°C
β	phase constant	rad/m
γ	propagation constant	rad/m
Γ	reflection coefficient	-
δ	skin depth	m
Δ	change, final minus initial	-
ε	permittivity	F/m
ε_0	free-space permittivity	8.854×10^{-12} F/m
ε_r	relative permittivity	-
η	efficiency	-
θ	phase angle	rad
κ	coupling coefficient	-
μ	permeability	H/m
μ_o	free-space permeability	1.2566×10^{-6} H/m
μ_r	relative permeability	-
ξ	ratio of radii	-
ρ	resistivity	$\Omega \cdot m$
σ	conductivity	S/m
ϕ	phase angle difference	rad
ϕ	impedance angle or power angle	rad
ϕ	current angle	rad
ϕ	phase angle difference	rad
ϕ_{pf}	power factor angle	rad
ω	armature angular speed	rad/s

Subscripts

ϕ	phase
0	zero sequence
0	characteristic
0	free space (vacuum)
0,o	initial (zero value)
1	positive sequence
1	primary
2	negative sequence
2	secondary
ab	a to b
AC	alternating current
avg, ave	average
bc	b to c
c	controls or critical
c	core
C	capacitive or capacitor
ca	c to a
Cu	copper
d	direct
DC	direct current
e	eddy current
e	equivalent
eff	effective
ext	external
E	generated voltage
f	final / frequency
fl	full load
g or *gen*	generator
h	hysteresis
i	imaginary
int	internal
I	current
l	line
l	per unit length

L	inductor or load
ll	line-to-line
ln	line to neutral
m	motor
m	maximum
m	mutual
max	maximum
n	neutral
nl	no load
O	origin
oc	open circuit
p	peak
p	phase
p	primary
pf	power factor
pri	primary
ps	primary to secondary
pu	per unit
q	quadrature
rms	root mean square
r	real
R	receiving end
R	resistance or resistive
s	secondary, source, synchronous
S	sending end
sc	short circuit
sec	secondary
t	terminal or total
thr	threshold
trans	transmission
V	potential difference between two points
w	wave
Z	impedance

TABLE OF CONTENTS

List of Figures

List of Tables

List of Equations

List of Examples

COURSE INTRODUCTION

The information is primarily from the author's books, Refs. [A] and [B] with the NESC information in Ref. [C]. The coverage of the NESC does not include end-users buildings—this is covered by the NEC, Ref. [D]. Information useful in many aspects of electric engineering may be found in [E] and [F] as well as the appendices. Reference [G] has detailed descriptions of analysis techniques. Reference [H] provides detailed engineering review. Reference [I] provides indepth explanation of the per-unit system often used in such engineering. Reference [J] covers many terms in EE with excellent definitions and explanations.

FUNDAMENTALS

Alternating waveforms have currents and voltages that vary with time in a regular and symmetrical manner. Waveform shapes include square, sawtooth, and triangular, along with many variations on these themes. However, for most applications in electrical engineering the variations are sinusoidal in time. In this book, unless otherwise specified, currents and voltages are sinusoidal.[2]

When sinusoidal, the waveform is nearly always referred to as AC—that is, alternating current—indicating that the current is produced by the application of a sinusoidal voltage. This means that the flow of electrons changes directions, unlike DC circuits, where the flow of electrons is unidirectional (though the magnitude can change in time).

A circuit is said to be in a steady-state condition if the current and voltage time variation is purely constant (DC) or purely sinusoidal (AC).[3] In this course, unless otherwise specified, all circuits are in a steady-state condition.

BENEFITS OF THREE-PHASE POWER

Three-phase energy distribution systems use fewer and smaller conductors and, therefore, are more efficient than multiple single-phase systems providing the same power. Three-phase motors provide a uniform torque, not a pulsating torque as do single-phase motors. Three-phase induction motors do not require additional starting windings or associated switches. When rectified, three-phase voltage has a smoother waveform and less ripple to be filtered.

---

[2] Nearly all periodic functions can be represented as the sum of sinusoidal functions, which simplifies the mathematics needed.
[3] Steady-state AC may have a DC offset.

Using three-phase power results in three currents that tend to cancel each other. Indeed three phases sum to zero in a linear balanced load. This allows the size of the neutral conductor to be reduced or eliminated. Since power transfer for three phases is constant and not pulsating as for a single- or two-phase machine, three-phase machine vibrations are reduced and bearing life is extended.[4] Three-phase systems can produce a magnetic field that rotates in a specified direction, simplifying the design of electric motors.

STANDARD NOTATION CONVENTIONS

Waveforms representative of the voltage in a power system are sinusoidal and have constant frequency, unless otherwise stated. Nonsinusoidal waveforms are dealt with by using transforms and transient analysis with varying frequencies. The most common and mathematically efficient way to represent such waveforms is with phasors.

Phasor representation is the primary tool for mathematical analysis of electrical systems with the voltage phasor as **V** and the current phasor as **I**, respectively. The magnitude, or the root-mean-square (rms) value, of a voltage or current phasor is given by V or I.[5] Maximum values carry a subscript, V_{max}. Impedance, Z, is technically not a phasor since it is steady with time. It is, however, a complex number and is treated identically to phasors with regard to notation. Subscripts are added to clarify where necessary.

A generated voltage is given denoted with *E* and a potential difference between two points with *V*. The primary (and preferred) method used to indicate potential difference is to use polarity signs, with the symbol “+” used to indicate the higher potential.

A secondary method is to use an arrow between the points, with the tail of the arrow indicating the lower potential.[6] A tertiary method is to use double-subscript notation, with the first subscript indicating the positive polarity.

[4] Using two or fewer, or more than three, will result in a situation where the total currents do not cancel one another in a balanced load. Further, there is additional expense involved.

[5] This course uses bold nomenclature to indicate magnitude. Some texts use $|\mathbf{V}|$ or $|\mathbf{I}|$, or no notation at all.

[6] This is based on the IEC standard for *Conventions Concerning Electric and Magnetic Circuit*, which is not always followed. Also, a curved arrow is used for potential and a straight arrow for current. Caution should be exercised when reviewing drawings.

The symbology for electromotive force, E, is similar to that used in the primary and secondary methods. However, if using the tertiary method, the first subscript will indicate negative polarity. Because of this difference, and to avoid confusion, this course uses a doubleheaded arrow for the potential difference and polarity markings.

Current-flow arrows always indicate the direction of positive current, and current flow is drawn in the same sense as the phase sequence. For example, in illustrations, current is shown as flowing into nodes A, B, and C of either wye or delta connections, and positive-sequence phase currents are shown as flowing from a to b for phase A; b to c for phase B; and c to a for phase C.

Conversion between various representative formats is often required to obtain a desired quantity. For example, voltage expressed as a function of time is given directly by Eq. 1. The maximum voltage is given as 169.7 V.[7]

Equation 1: Voltage as a Function of Time

$$v(t) = \left(169.7 \text{ V}\right)\sin\left(\omega t + 30^\circ\right)$$

The magnitude (rms) value of the voltage is given by Eq. 2. Substituting 169.7 V for V_p, V_{rms} is equal to 120 V.

Equation 2: RMS Voltage

$$V_{\text{rms}} = \frac{V_p}{\sqrt{2}}$$

The next few equations show an example of using Euler's identity is to change the time representation of the voltage shown in Eq. 3 into an effective (rms) phasor representation.

Equation 3: Maximum Voltage Representation as a Function of Time

$$v(t) = \left(169.7 \text{ V}\right)\sin\left(\omega t + 30^\circ\right)$$

[7] The forms of Eq. 1 and Eq. 3 are not strictly correct mathematically, but this form is commonly seen in electrical engineering practice for convenience. The time-dependent part, ωt, has the units of radians, and the phase angle, 30°, has units of degrees.

Euler's identity is given by Eq. 4.

Equation 4: Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Either the cosine or the sine can be used as the reference.[8] In this case, the voltage is given in Eq. 3. Writing the voltage in terms of Euler's identity gives the following.

Equation 5: Maximum Voltage in Euler's Identity Form

$$v(t) = \mathrm{Re}\left(\left(169.7\ \mathrm{V}\right)e^{j(\omega t + 30^\circ)}\right)$$

Separate the voltage into nonrotating and rotating portions. Additionally, separate the maximum value into its rms (effective) value by dividing the maximum by the square root of 2, gives Eq. 6.

Equation 6: Effective Voltage with Non-Rotating/Rotating Forms

$$v(t) = \mathrm{Re}\left(\left(\frac{169.7\ \mathrm{V}}{\sqrt{2}}\right)e^{j30^\circ}e^{j\omega t}\right)$$

Since the rotating portion and the square root of two are common to all the electrical quantities associated with this voltage (consistent with the assumption that only one frequency is present during steady-state conditions, and that effective values are being used), the phasor notation can be written directly from Eq. 6, shown in Eq. 7.

Equation 7: Phasor Voltage Notation

$$\mathbf{V} = \left(120\ \mathrm{V}\right)e^{j30^\circ} = \left(120\ \mathrm{V}\right)e^{j\frac{\pi}{6}} = 120\ \mathrm{V}\angle 30^\circ$$

---

[8] The sine reference is often used since the value of the function at $t = 0$ is zero, which correlates with standard graphing techniques. The cosine reference is also used because it represents the real part of Euler's identity and avoids the j term. Many national and international electrical standards also use cosine as the reference. This course will emphasize the cosine reference unless graphing is involved. (However, both the sine and cosine reference are still in use; only the phase angle changes.) Care should be taken to ensure the desired reference is used, which can be determined from the equation provided. For example, the voltage source is given as a cosine function; therefore, that is the desired reference.

The voltage is given as V = 120 V∠30°. The angle was carried as 30° throughout, but any calculation using the exponential would need to be accomplished using the radian value, in this case π/6.[9]

SINGLE-SUBSCRIPT NOTATION

Single-subscript notation may be used for DC circuits, AC single-phase circuits, and three-phase circuits where confusion cannot occur (for example, where only power calculations occur and directions of currents or polarity of voltages are not required), and for a single phase of a three-phase balanced circuit.

In Fig. 1, a single phase of a three-phase circuit is shown. As long as the circuit is balanced, calculations are valid for all three phases. Furthermore, the power calculation on this circuit when multiplied by three results in the total power for the three-phase circuit.

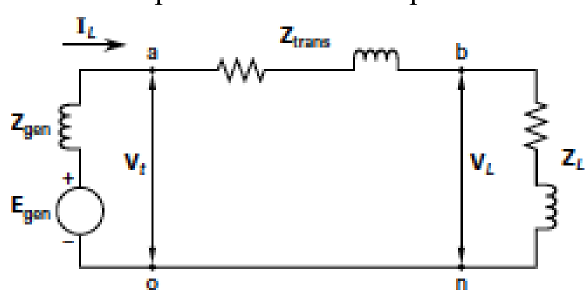


Figure 1: Single Phase of a Three-Phase Circuit

The polarity marks and the direction of the current are required in order to indicate these voltages are phasors. The polarity marks indicate the potential and current flow during the positive half-cycle of the source, in this case, E_g. The potential and current flow change direction in the negative half-cycle, but the power flow direction does not.

The load current, I_L, flows through both the load and the wires, also known as transmission/distribution lines. The voltage drop across the transmission line is given by Eq. 8.

Equation 8: Transmission Line Voltage Drop

[9] The other item to take care with is whether maximum or effective values are being used. If not stated it's generally safe to assume effective values, especially if a standard voltage is shown, such as 120 V. If a trigonometric form is being used, it's likely the voltage is the maximum value.

$$\begin{aligned}\mathbf{V}_{trans} &= I_{trans}Z_{trans} = I_L Z_{trans} \\ &= \left(\frac{\mathbf{V}_t - \mathbf{V}_L}{\mathbf{Z}_{trans}}\right)\mathbf{Z}_{trans} \\ &= \mathbf{V}_t - \mathbf{V}_L\end{aligned}$$

Using the nodes shown in Fig. 1 instead of polarity marks and arrows, and noting that the voltages at nodes a and b can be assumed to be compared to a *reference node* at o (for origin), which could also be called the *neutral node*, n, results in the following instantaneous and phasor single-subscript voltages. Single-subscript notation is shown in Eqs. 9-13.

Equation 9: Instantaneous Voltage A

$$v_a = v_t$$

Equation 10: Instantaneous Voltage B

$$v_b = v_L$$

Equation 11: Phasor Voltage A

$$\mathbf{V}_a = \mathbf{V}_t$$

Equation 12: Phasor Voltage B

$$\mathbf{V}_b = \mathbf{V}_L$$

Equation 13: Transmission Line Phasor Voltage

$$\mathbf{V}_{trans} = \mathbf{V}_t - \mathbf{V}_L$$

DOUBLE-SUBSCRIPT NOTATION

Polarity marks and current arrows are not required if double-script notation is used. Three-phase circuits are represented using subscripts to clarify relationships. For example, Eq. 13 can be rewritten using the doublesubscript notation as Eq. 14.

Equation 14: Double-Subscript Notation

$$\mathbf{V}_{trans} = \mathbf{V}_{ab}$$

In single-subscript notation, when defining a current, the arrow represents the direction of positive current flow. When defining a voltage, a reference node is used, and the voltage at one node is compared to the voltage at another node, though that node was not acknowledged in the representation itself. In double-subscript notation, the voltage of the first subscript is given with respect to the second subscript (i.e., the reference node).

Therefore, the single-subscript load current, $\mathbf{I}_L$, in Fig. 1 can be represented as $\mathbf{I}_{ab}$. The single-subscript voltage $\mathbf{V}_{trans}$ can be represented as $\mathbf{V}_{ab}$. The impedance between nodes a and b can be represented as

Equation 15: Phasor Transforms of Sinusoids

$$\mathbf{Z}_{trans} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_{ab}} = \mathbf{Z}_{ab}$$

GENERATION OF THREE-PHASE POTENTIAL

The symbolic representation of an AC generator that produces three equal sinusoidal voltages is shown in Fig. 2(a). The generated voltage is known as the *phase voltage*, V_p, or *coil voltage*. (Three-phase voltages are almost always stated as effective values.) Primarily because of the location of the windings, the three sinusoids are 120° apart in phase as shown in Fig. 2(b). *If V_a is chosen as the reference voltage*, then Eq. 16 through Eq. 18 represent the phasor forms of the three sinusoids.[10] *At any moment, the vector sum of these three voltages is zero*.

Equation 16: 3-Phase Voltage A

$$\mathbf{V}_a = V_p \angle 0^\circ$$

Equation 17: 3-Phase Voltage B

$$\mathbf{V}_b = V_p \angle -120^\circ$$

Equation 18: 3-Phase Voltage C

$$\mathbf{V}_c = V_p \angle -240^\circ$$

---

[10] The negative sign can be confusing. After all, the are rotating in the counter-clockwise direction. But recall, with phasor "a" as the reference, at 0°, to get to "b" one needs to go in the negative direction. The phasor "b" is the next one to pass through the reference point of zero, but it will be –120° behind phasor "a". In other words, phase "a" passes the *x*-axis and continues counterclockwise. When phase "b" passes the *x*-axis, phase "a" is 120° ahead of that point. Or, from the phase "a" point of reference, phase "b" is –120° behind phase "a".

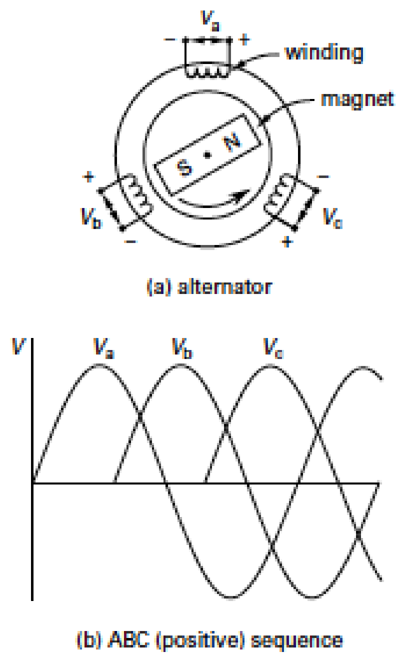


Figure 2: Three-Phase Rotation

Equation Eq. 16 through Eq. 18 define an ABC or *positive sequence*. That is, $\mathbf{V}_a$ reaches its peak before $\mathbf{V}_b$, and $\mathbf{V}_b$ peaks before $\mathbf{V}_c$. With a CBA (also written as ACB) or *negative sequence*, obtained by rotating the field magnet in the opposite direction, the order of the delivered sinusoids is reversed (i.e., $\mathbf{V}_c$, $\mathbf{V}_b$, $\mathbf{V}_a$).
Although a six-conductor transmission line could be used to transmit the three voltages, it is more efficient to interconnect the windings. The two methods are commonly referred to as delta (mesh) and wye (star) connections.

Figure 3(a) illustrates delta source connections. The voltage across any two of the lines is known as the line voltage (system voltage) and is equal to the phase voltage. Any of the coils can be selected as the reference as long as the sequence is maintained. For a positive (ABC) sequence,

Equation 19: 3-Phase Delta Voltage CA

$$\mathbf{V}_{CA} = V_p \angle 0^\circ$$

Equation 20: 3-Phase Delta Voltage AB

$$\mathbf{V}_{AB} = V_p \angle -120^\circ$$

Equation 21: 3-Phase Delta Voltage BC

$$\mathbf{V}_{BC} = V_p \angle -240^\circ$$

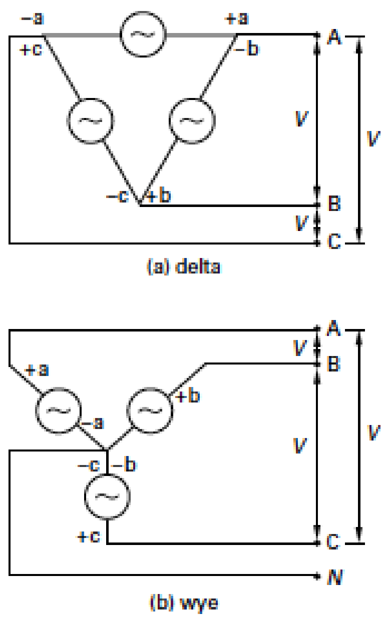


Figure 3: Delta and Wye Source Connections

Wye-connected sources are illustrated in Fig. 3(b). While the *line-to-neutral voltages* are equal to the phase voltage, the line voltages are greater by an amount √3 times the phase voltage. The *grounded wire* (neutral) is needed to carry current only if the system is unbalanced.[11] For an ABC sequence, the line voltages are

Equation 22: 3-Phase Wye Voltage AB

$$\mathbf{V}_{AB} = \sqrt{3} V_p \angle 30^\circ$$

Equation 23: 3-Phase Wye Voltage BC

$$\mathbf{V}_{BC} = \sqrt{3} V_p \angle -90^\circ$$

[11] The neutral wire is usually kept to provide for a minor imbalance.

Equation 24: 3-Phase Wye Voltage CA

$$\mathbf{V}_{CA} = \sqrt{3}V_p\angle -210^\circ$$

Although the magnitude of the line voltage depends on whether the generator coils are delta- or wye-connected, each connection results in three equal sinusoidal voltages, each 120° out of phase with the others.

DISTRIBUTION SYSTEMS

Three-phase power is delivered by three-wire and four-wire systems. A *four-wire system* consists of three power conductors and a neutral conductor. A *three-wire system* contains only the three power conductors.

Utility power distribution starts with generation. The generator is connected through step-up *subtransmission transformers* that supply *transmission lines*. The actual transmission line voltage depends on the distance between the subtransmission transformers and the user.

Distribution *substation transformers* reduce the voltage from the transmission line level to approximately 35 kV. The *primary distribution system* delivers power to distribution transformers that reduce voltage still further, to between 120 V and 600 V.

BALANCED LOADS

Three impedances are required to fully load a three-phase voltage source. The impedances in a three-phase system are *balanced* when they are identical in magnitude and angle. The voltages and line currents, as well as the real, apparent, and reactive powers, are all identical in a balanced system. Also, the power factor is the same for each phase. Therefore, balanced systems can be analyzed on a per-phase basis. Such calculations are known as *one-line analyses*.

Figure 4 illustrates the vector diagram for a balanced delta three-phase system. The phase voltages, **V**, are separated by 120° phase angles, as are the phase currents, **I**. The phase difference angle, ϕ, between a phase voltage and its respective phase current depends on the phase impedance. With delta-connected balanced loads, the phase and line currents differ in phase by 30°.

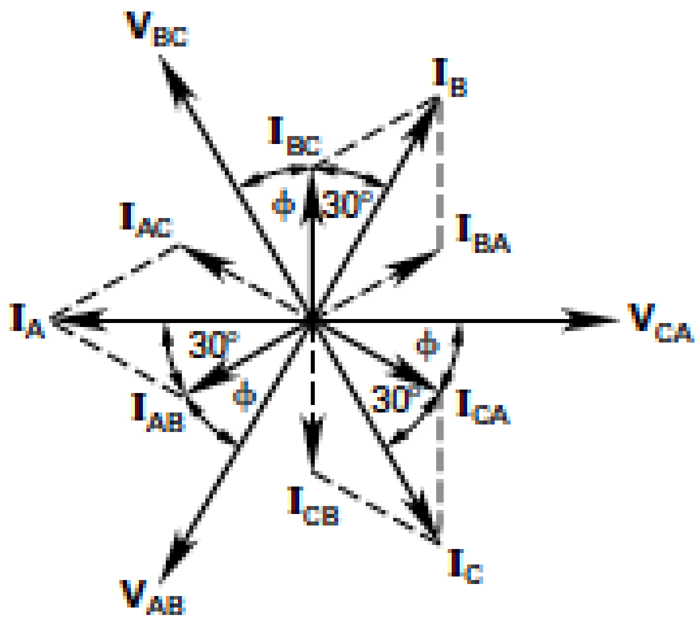


Figure 4: Positive Sequence (ABC) Balanced Delta Load Vector Diagram

DELTA-CONNECTED LOADS

Figure 5 illustrates delta-connected loads.

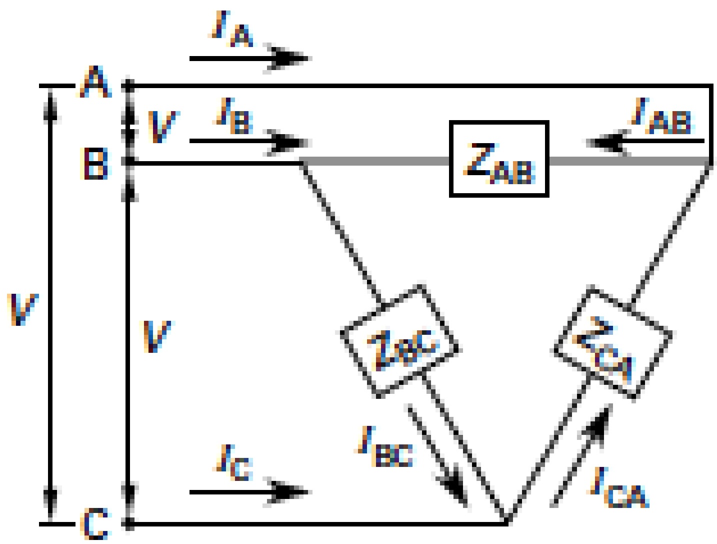


Figure 5: Delta-Connected Loads

The *phase currents* for a balanced system are calculated from the line voltage (same as the phase voltage). (Refer to Figs. 4 & 5.) For a positive (ABC) sequence,

Equation 25: Balanced System Phase Current AB

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{V\angle -120^\circ}{Z\angle \phi} = \frac{V}{Z}\angle -120^\circ - \phi$$

Equation 26: Balanced System Phase Current BC

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{V}{Z}\angle -240^\circ - \phi$$

Equation 27: Balance System Phase Current CA

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{V}{Z}\angle -\phi$$

The *line currents* are not the same as the phase currents but are √3 times the phase current and displaced −30° in phase from the phase current.

Equation 28: Balanced System Line Current AB

$$|\mathbf{I}_A| = |\mathbf{I}_{AB} - \mathbf{I}_{CA}| = \sqrt{3}I_{AB}$$

Equation 29 Balanced System Line Current BC

$$|\mathbf{I}_B| = |\mathbf{I}_{BC} - \mathbf{I}_{AB}| = \sqrt{3}I_{BC}$$

Equation 30: Balanced System Line Current CA

$$|\mathbf{I}_C| = |\mathbf{I}_{CA} - \mathbf{I}_{BC}| = \sqrt{3}I_{CA}$$

Equation 31: Balanced System Line Current Magnitudes

$$I_A = I_B = I_C \quad \textbf{[balanced]}$$

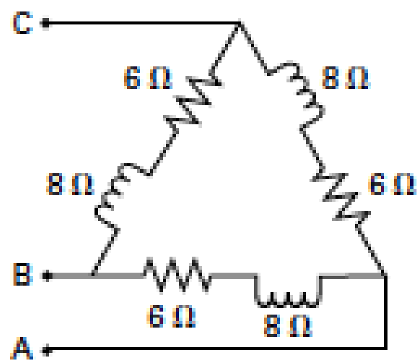
Each impedance in a balanced system dissipates the same real *phase power*, P_p. The total power dissipated is three times the phase power. (This is the same as for wye-connected loads.)

Equation 32: Delta Power Formulas

$$\begin{aligned} P_{\text{total}} &= 3P_{\text{phase}} \\ &= 3V_p I_p \cos|\phi| = 3V_p I_p \text{pf} \\ &= \sqrt{3}VI\cos|\phi| = \sqrt{3}VI\text{pf} \end{aligned}$$

Example 1

Three identical impedances are connected in delta across a three-phase system with 240 V (rms) line voltages in an ABC sequence. Find the (a) phase current $\mathbf{I}_{AB}$, (b) phase real power, P_p, (c) line current $\mathbf{I}_B$, and (d) total real power.



Solution

(a) The phase impedance is

$$\begin{aligned}\mathbf{Z}_p &= \sqrt{R^2 + X_L^2}\angle\arctan\frac{X}{R}\\ &= \sqrt{(6\ \Omega)^2 + (8\ \Omega)^2}\arctan\frac{8\ \Omega}{6\ \Omega}\\ &= 10\ \Omega\angle 53.13^\circ\end{aligned}$$

Therefore, the phase current is calculated from

$$\begin{aligned}\mathbf{I}_{AB} &= \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}}\\ &= \frac{240\ \text{V}\angle -120^\circ}{10\ \Omega\angle 53.13^\circ}\\ &= 24\ \text{A}\angle -173.13^\circ\end{aligned}$$

(b) The average phase power is

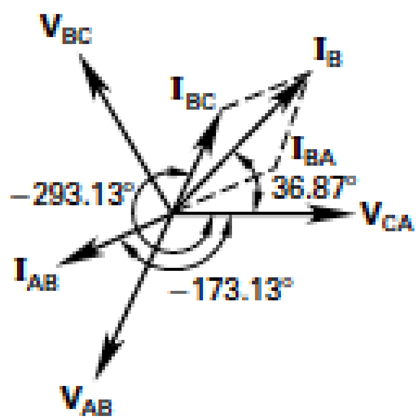
$$P_p = I_p^2 R$$
$$= (24\ \text{A})^2 (6\ \Omega)$$
$$= 3456\ \text{W}$$

(c) The phase current I_{BC} contributes to the line current.

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{240\ \text{V}\angle -240^\circ}{10\ \Omega\angle 53.13^\circ} = 24\ \text{A}\angle -293.13^\circ$$

$$\mathbf{I}_B = \mathbf{I}_{BC} - \mathbf{I}_{AB} = 24\ \text{A}\angle -293.13^\circ - 24\ \text{A}\angle -173.13^\circ$$
$$= 41.57\ \text{A}\angle 36.87^\circ$$

Graphically, it looks like so,



(d) The total power is three times the phase power.

$$P_t = 3P_p = (3)(3456\ \text{W})$$
$$= 10{,}368\ \text{W}$$

WYE-CONNECTED LOADS

Figure 6 illustrates three equal impedances connected in a wye configuration. The line and phase currents are equal. However, the phase voltage is less than the line voltage because more than a single phase is connected across two lines. The line and phase current equations follow.

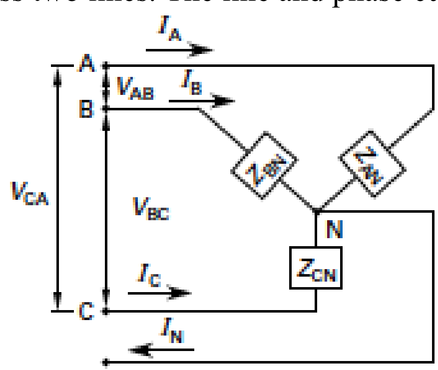


Figure 6: Wye Connected Loads

Equation 33: Wye Current A

$$\mathbf{I}_A = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_{AN}} = \frac{\left(\frac{\mathbf{V}_{AB}}{\sqrt{3}}\right)}{\mathbf{Z}_{AN}} = \frac{\mathbf{V}_{AB}}{\sqrt{3}\mathbf{Z}_{AN}}$$

Equation 34: Wye Current B

$$\mathbf{I}_B = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_{BN}} = \frac{\left(\frac{\mathbf{V}_{BC}}{\sqrt{3}}\right)}{\mathbf{Z}_{BN}} = \frac{\mathbf{V}_{BC}}{\sqrt{3}\mathbf{Z}_{BN}}$$

Equation 35: Wye Current C

$$\mathbf{I}_C = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_{CN}} = \frac{\left(\frac{\mathbf{V}_{CA}}{\sqrt{3}}\right)}{\mathbf{Z}_{CN}} = \frac{\mathbf{V}_{CA}}{\sqrt{3}\mathbf{Z}_{CN}}$$

Equation 36: Wye Neutral Current

$$\mathbf{I}_N = 0 \quad \text{[balanced]}$$

The total power dissipated in a balanced wye-connected system is three times the phase power. (This is the same as for the delta connection.)

Equation 37: Wye Power Formulas

$$
\begin{aligned}
P_{\text{total}} &= 3P_{\text{phase}} \\
&= 3V_p I_p \cos|\phi| = 3V_p I_p \text{pf} \\
&= \sqrt{3} VI \cos|\phi| = \sqrt{3} VI \text{pf}
\end{aligned}
$$

Example 2

A three-phase, 480 V, 250 hp motor has an efficiency of 94% and a power factor of 90%. What is the line current?

Solution

It does not matter whether the motor's windings are delta- or wye-connected. From Eq. 37, the line current is

$$P_t = \eta \sqrt{3} VI \cos\phi$$

$$
\begin{aligned}
I &= \frac{P_t}{\eta \sqrt{3} V \cos\phi} \\
&= \frac{(250 \text{ hp})\left(745.7 \frac{\text{W}}{\text{hp}}\right)}{(0.94)\left(\sqrt{3}\right)(480 \text{ V})(0.9)} \\
&= 265 \text{ A}
\end{aligned}
$$

DELTA-WYE CONVERSIONS

It is occasionally convenient to convert a delta system to a wye system and vice versa. If need be, the equations in Ref [B], used to convert between delta and wye resistor networks, are applicable if impedances are substituted for resistances.

PER-UNIT CALCULATIONS

Power systems use a multitude of voltages ranging from very high (35,000 V) to very low (120 V). To simplify analysis, a per-unit system is employed that scales the sections of the distribution system in accordance with the voltage ratios of the transformers. The per-unit system is given mathematically by

Equation 38: Per Unit System Defined

$$\text{per unit} = \frac{\text{actual}}{\text{base}} = \frac{\text{percent}}{100}$$

For a three-phase system, the usual bases are the line voltage (in V) and the total (three-phase) apparent power (in VA) ratings. From these two bases, the base current and impedance can be calculated. Line and phase values may differ in three-phase systems. The relationship between the phase and line quantities for the bases is

Equation 39: Phase and Line Relationship in Voltage Base

$$V_p = \frac{V_l}{\sqrt{3}}$$

Equation 40: Phase and Line Relationship in Apparent Power Base

$$S_p = \frac{S_t}{3}$$

The line-to-phase voltage conversion is necessary in a wye connection only. In a delta connection, the line and phase voltages are equal. Care should be taken when using phase and line quantities because the symbols vary. Phase values are represented here with the subscript *p*. However, phase values are also given with subscripts ϕ and l_n (line-to-neutral). Line values are given with the subscript *t* (total) here. However, line values are also given with subscripts *ll* (line-to-line), *l* (line), 3ϕ, or with no subscript. The per-unit system is represented by Eq. 41 through Eq. 46. It is presented elsewhere as phase bases for use with single-phase systems. Conversion to a three-phase base, which uses line and total quantities, is accomplished with Eq. 39 and Eq. 40 and is indicated by the subscript 3ϕ in Eq. 41 through Eq. 46.

Equation 41: Per Unit Apparent Power Base

$$S_{\text{base}} = S_p = \left(\frac{S}{3}\right)_{3\phi}$$

Equation 42: Per Unit Voltage Base

$$V_{\text{base}} = V_p = \left(\frac{V}{\sqrt{3}}\right)_{3\phi}$$

Equation 43: Per Unit Current Base

$$I_{\text{base}} = \frac{S_{\text{base}}}{V_{\text{base}}} = \frac{S_p}{V_p} = \left(\frac{S}{\sqrt{3}V}\right)_{3\phi}$$

Equation 44: Per Unit Impedance Base

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_p^2}{S_p} = \left(\frac{V^2}{S}\right)_{3\phi} = \left(\frac{V}{\sqrt{3}I}\right)_{3\phi}$$

Equation 45: Per Unit Real Power Base

$$P_{\text{base}} = P_p = \left(\frac{P}{3}\right)_{3\phi}$$

Equation 46: Per Unit Reactive Power Base

$$Q_{\text{base}} = Q_p = \left(\frac{Q}{3}\right)_{3\phi}$$

The per-unit values are as follows.

Equation 47: Per Unit Current

$$I_{\text{pu}} = \frac{I_{\text{actual}}}{I_{\text{base}}}$$

Equation 48: Per Unit Voltage

$$V_{\text{pu}} = \frac{V_{\text{actual}}}{V_{\text{base}}}$$

Equation 49: Per Unit Impedance

$$Z_{\text{pu}} = \frac{Z_{\text{actual}}}{Z_{\text{base}}}$$

Equation 50: Per Unit Real Power

$$P_{\text{pu}} = \frac{P_{\text{actual}}}{P_{\text{base}}}$$

Equation 51: Per Unit Reactive Power

$$Q_{\text{pu}} = \frac{Q_{\text{actual}}}{Q_{\text{base}}}$$

Ohm's law and other circuit analysis methods can be used with the per-unit quantities.

Equation 52: Per Unit Ohm's Law

$$V_{\text{pu}} = I_{\text{pu}} Z_{\text{pu}}$$

All values in a given portion of a power system must be expressed using the same base. The general method for converting from one generic per-unit value, call it χ, to another in a different base is[12]

Equation 53: Per Unit Base Change

$$\chi_{\text{pu,new}} = \chi_{\text{pu,old}} \left(\frac{\chi_{\text{base,old}}}{\chi_{\text{base,new}}} \right)$$

The impedance per-unit value conversion is as follows.

---

[12] This doesn't look like it cancels correctly; but this is the correct equation. The impedance per-unit transformation doesn't fit neatly into this form, see the next equation.

Equation 54: Impedance Base Change

$$Z_{\text{pu,new}} = Z_{\text{pu,old}}\left(\frac{V_{\text{base,old}}}{V_{\text{base,new}}}\right)^2\left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}}\right)$$

Values of per-unit quantities are initially given or calculated using the rated values of the electrical components (that is, the rating of the generator, transformer, or motor). The per-unit value for a transformer is the same whether referenced to either the low or high side. (Example 4 provides evidence for this assertion.) The purpose of Eq. 53 and Eq. 54 is to adjust to the base value used by the entire system.

When changing from per-unit values to actual values or vice versa (for example, transmission line impedance conversion), the *base voltage used must be the voltage value present in the given section of the electrical distribution system for which the results are desired*.

Example 3

A wye-connected three-phase device is rated to draw a total of 300 kVA when connected to a line-to-line voltage of 15 kV. The device's per-unit impedance is 0.1414 + *j*0.9900. What is the actual impedance?

Solution

The bases are as follows.

$$V_{\text{base}} = V_p = \frac{V}{\sqrt{3}} = \frac{15\ \text{kV}}{\sqrt{3}} = 8.66\ \text{kV}$$

$$\text{S}_{\text{base}} = S_{\text{phase}} = \frac{S}{3} = \frac{300\ \text{kVA}}{3} = 100\ \text{kVA}$$

The base current follows.

$$I_{\text{base}} = \frac{S_{\text{base}}}{V_{\text{base}}} = \frac{100\ \text{kVA}}{8.66\ \text{kV}} = 11.55\ \text{A}$$

The base impedance follows.

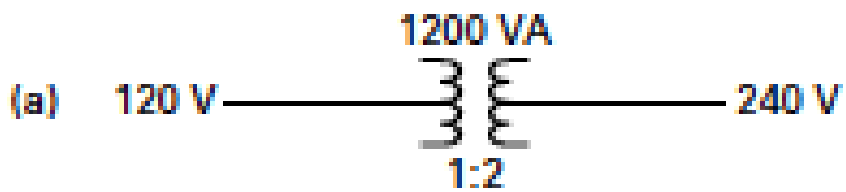
$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{8660\ \text{V}}{11.55\ \text{A}} = 750\ \Omega$$

From Eq. 49, the actual impedance is calculated as

$$Z_{\text{actual}} = Z_{pu} Z_{\text{base}} = \left(0.1414 + j0.9900\right)\left(750\ \Omega\right) = 106.1 + j742.5\ \Omega$$

Example 4

Review the one-line diagram for a three-phase generatorreactor system in the figure shown. The actual impedance on the low voltage side is $j4.0\ \Omega$. What is the ratio of the low-side to the high-side per-unit impedance?



Solution

Given the values in the figure, and noting that the impedance on the high voltage side of the transformer must be 16.0 Ω, the quantities can be calculated, as shown in Table 1. The per-unit values are in the final row.[13]

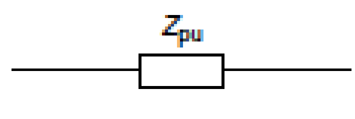
[13] This table is of significant importance. It shows how the per unit values on either side of the transformer are identical allowing it to be shown as a single impedance value and allowing expedited (i.e., hand calculations) for short circuit issues as fairly accurate first order results.

Table 1: Per-Unit Equivalency

quantity	low voltage side	high voltage side
S_{base}	1200 VA	1200 VA
V_{base}	120 V	240 V
I_{base}	$\frac{S_{\text{base}}}{V_{\text{base}}} = \frac{1200\ \text{VA}}{120\ \text{V}} = 10\ \text{A}$	$\frac{S_{\text{base}}}{V_{\text{base}}} = \frac{1200\ \text{VA}}{240\ \text{V}} = 5\ \text{A}$
Z_{base}	$\frac{V^2_{\text{base}}}{S_{\text{base}}} = \frac{(120\ \text{V})^2}{1200\ \text{VA}} = 12\ \Omega$	$\frac{\mathbf{V}^2_{\text{base}}}{S_{\text{base}}} = \frac{(240\ \text{V})^2}{1200\ \text{VA}} = 48\ \Omega$
$\mathbf{Z}_{\text{pu}}$	$\frac{\mathbf{Z}_{\text{low side}}}{Z_{\text{base}}} = \frac{j4.0\ \Omega}{12\ \Omega} = j0.33\ \text{pu}$	$\frac{\mathbf{Z}_{\text{high side}}}{Z_{\text{base}}} = \frac{j16.0\ \Omega}{48\ \Omega} = j0.33\ \text{pu}$

The ratio of the low-side per-unit value to the high-side per-unit value is 1:1. This indicates that a model of the per-unit impedance for the transformer (single phase) is

(b)



This can be represented in a one-line diagram as follows.

(c)

Because line quantities are used, this model also works for three-phase systems.

UNBALANCED LOADS

The three-phase loads are unequal in an unbalanced system. A fourth conductor, the neutral conductor, is required for the line voltages to be constant. Such a system is known as a *four-wire system*. Without the neutral conductor (i.e., in a *three-wire system*), the common point of the load connections is not at zero potential and the voltages across the three impedances vary from the line-to-neutral voltage. The voltage at the common point is known as the *displacement neutral voltage*.

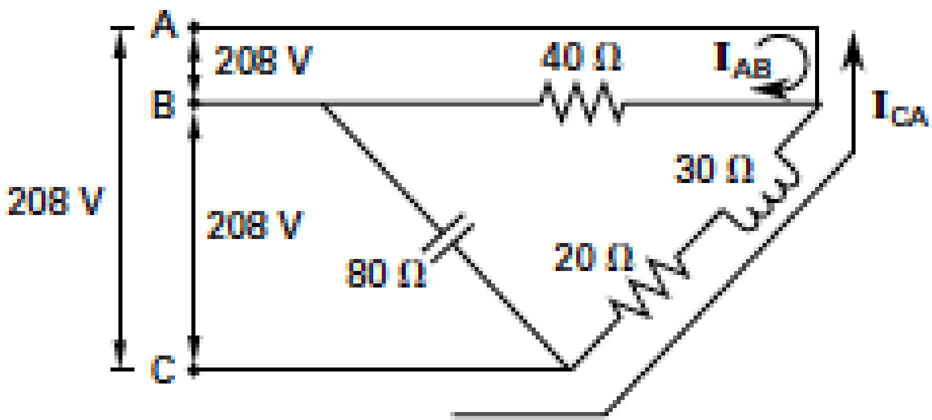
Regardless of whether the line voltages are equal, the line currents are not the same nor do they have a 120° phase difference. Unbalanced systems can be evaluated by computing the phase currents and then applying Kirchhoff's current law (in vector form) to obtain the line currents. The neutral current is

Equation 55: Neutral Current

$$\mathbf{I}_N = -\left(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_c\right)$$

Example 5

Unequal impedances are connected to a 208 V (rms) three-phase system as shown. The sequence is ABC with ∠VCA = 0°. (a) What is the line current $\mathbf{I}_A$? (b) What is the total real power dissipated?



Solution

(a) First, calculate the phase current $\mathbf{I}_{AB}$.

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{208\text{ V}\angle -120^\circ}{40\ \Omega\angle 0^\circ} = 5.20\text{ A}\angle -120^\circ$$

The phase impedance $\mathbf{Z}_{CA}$ is

$$\mathbf{Z}_{CA} = \sqrt{\left(R^2 + X_L^2\right)}\angle\arctan\frac{X_L}{R} = \sqrt{\left(20\ \Omega\right)^2 + \left(30\ \Omega\right)^2}\angle\arctan\frac{30\ \Omega}{20\ \Omega} = 30.06\ \Omega\angle 56.31^\circ$$

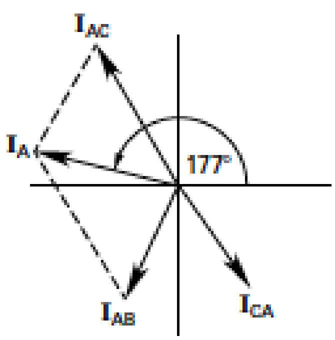
Next, the phase current $\mathbf{I}_{CA}$ is as follows.

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{208\ \text{V}\angle 0^\circ}{36.06\ \Omega\angle 56.31^\circ} = 5.77\ \text{A}\angle -56.31^\circ$$

The line current is

$$\begin{aligned}\mathbf{I}_{A} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} + \mathbf{I}_{AC} \\ &= 5.20\ \text{A}\angle -120^\circ + 5.77\ \text{A}\angle 123.69^\circ \\ &= 5.81\ \text{A}\angle 177^\circ\end{aligned}$$

On a diagram, it appears thus.



(b) The total power dissipated is

$$\begin{aligned}P &= \Sigma I^2 R \\ &= (5.20\ \text{A})^2(40\ \Omega) + (5.77\ \text{A})^2(20\ \Omega) \\ &= \mathbf{1747\ W}\end{aligned}$$

THREE-PHASE TRANSFORMERS

Transformer banks for three-phase systems have three primary and three secondary windings. Each primary-secondary set is referred to as a *transformer*, as distinguished from the *bank*. Each side can be connected in a delta or wye configuration, making a total of four possible transformer configurations (delta-delta, delta-wye, wye-delta, and wye-wye) as shown in Fig. 7. The turns ratio, *a*, is the same for each winding. Each transformer provides one-third of the total kVA rating, regardless of connection configuration. However, as the figure shows, the secondary voltages and currents depend on the configuration.

Equation 56: Turns Ratio

$$a = \frac{N_{\text{pri}}}{N_{\text{sec}}}$$

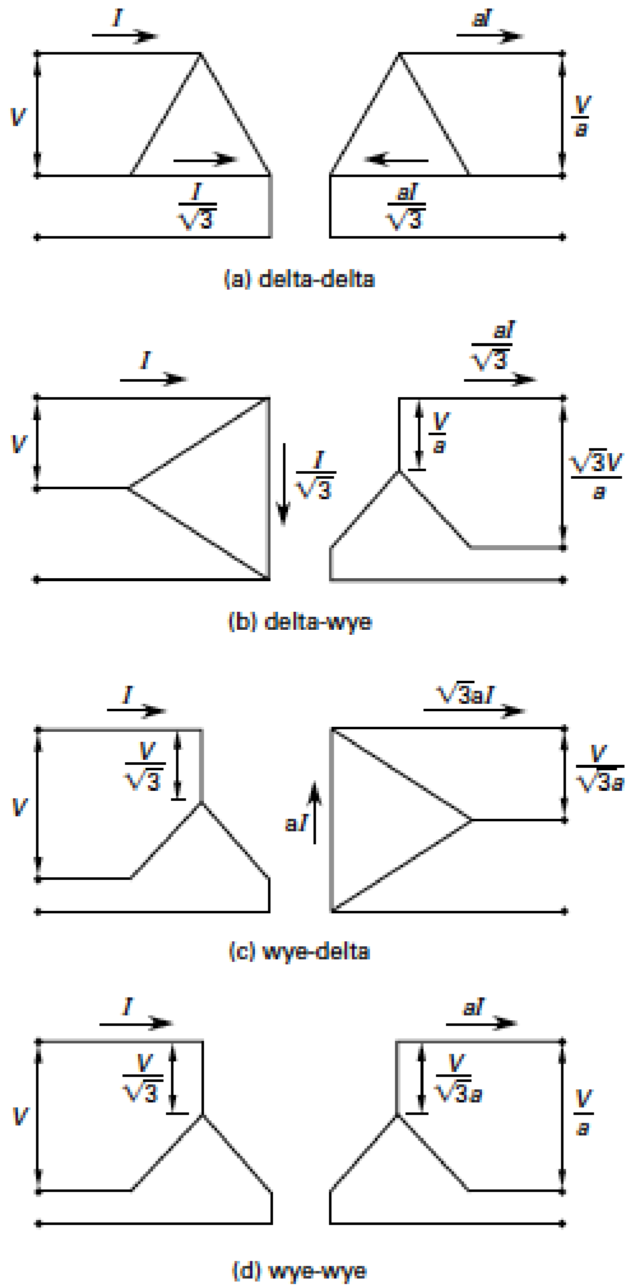


Figure 7: Transformer Configurations

Example 6

A 240 V (rms) three-phase system drawing 1200 kVA is supplied by a 2400 V (primary-side) transformer bank. Each transformer is connected in a wye-delta (primary-secondary) configuration.

What are the (a) ratio of transformation, (b) high-side and low-side winding voltages, (c) high-side and low-side winding currents, and (d) kVA rating?

Solution

(a) The primary line voltage is 2400 V serving a secondary voltage of 240 V. From this, and paying attention to the wye-delta configuration, one may calculate the turns ratio.

$$a = \frac{V_{\text{phase,pri}}}{V_{\text{phase,sec}}} = \frac{\frac{V_{\text{line,pri}}}{\sqrt{3}}}{V_{\text{line,sec}}}$$

$$= \frac{V_{\text{line,pri}}}{V_{\text{line,sec}}\sqrt{3}} = \frac{2400\ \text{V}}{(240\ \text{V})(\sqrt{3})} = 5.774$$

(b) The winding (phase) voltages are

$$V_{\text{high}} = \frac{V}{\sqrt{3}} = \frac{2400\ \text{V}}{\sqrt{3}} = 1386\ \text{V}$$

$$\text{V}_{\text{low}} = 240\ \text{V} \quad \text{[given]}$$

(c) The winding (phase) currents are

$$I_{\text{high}} = \frac{S_p}{V_p} = \frac{S_t/3}{V/\sqrt{3}} = \frac{\sqrt{3}S_t}{3V}$$
$$= \frac{\sqrt{3}\left(1200 \times 10^3 \text{ VA}\right)}{(3)(2400 \text{ V})}$$
$$= 288.7 \text{ A}$$

$$\text{I}_{\text{low}} = aI_{\text{high}}$$
$$= (5.774)(288.7 \text{ A})$$
$$= 1667 \text{ A}$$

(d) The transformer winding kVA rating (per phase) is

$$S_{\text{high}} = I_{\text{high}}V_{\text{high}} = (288.7 \text{ A})(1386 \text{ V})$$
$$= 4 \times 10^5 \text{ VA} = 40 \times 10^4 \text{ VA} = 400 \times 10^3 \text{ VA}$$
$$= 400 \text{ kVA}$$

FAULTS AND FAULT CURRENT

A *fault* is an unwanted connection (i.e., a short circuit) between a line and ground or another line. Although the fault current is usually very high before circuit breakers trip, it is not infinite, because the transformers and transmission line have finite impedance up to the fault point. If the line impedance is known, the fault current can be found by Ohm's law.[14]

Equation 57: Fault Current

$$V = I_{\text{fault}}Z$$

[14] Equation 61 ignores the transient (DC) current component and the relatively small current flowing in the wire before the fault occurs. Although the fault current has a transient component, it dies out so quickly that it is insignificant.

The voltage level during a transient will change, though using the rated value can provide a worst-case condition. Impedance levels change during the three stages of any fault condition. So Eq. 61 can, and should, be modified to provide more detailed analysis. Details are given in a future course on *Fault Analysis* and Ref [B]. Additionally, the IEEE Standard 141, titled *IEEE Recommended Practice for Electric Power Distribution for Industrial Plants*, provides detailed information on short-circuit analysis techniques.

REFERENCES

A. Camara, John A. *Electrical Engineering Reference Manual.* Belmont, CA: PPI, 2009.
B. Camara, John A. *PE Power Reference Manual*. Belmont, CA: PPI (Kaplan), 2021.
C. Marne, David J., and John A. Palmer. *National Electrical Safety Code®* (NESC®) *2023 Handbook*. New York: McGraw Hill, 2023.
D. Earley, Mark, ed. *NFPA 70, National Electrical Code Handbook*. Quincy, Massachusetts: NFPA, 2020.
E. IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.
F. IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE.
G. Grainger, John J., and William Stevenson, Jr. *Power System Analysis*. New York, McGraw Hill, 1994.
H. Grigsby, Leonard L., ed. *Electric Power Generation, Transmission, and Distribution.* Boca, Raton, FL: CRC Press, 2012
I. Wikipedia, The Free Encyclopedia (2024). "Per-Unit System". https://en.wikipedia.org/wiki/Per-unit_system (accessed FEB 2024).
J. Parker, Sybil P., editor in chief. *McGraw-Hill Dictionary of Scientific and Technical Terms,* 5th ed. New York, McGraw-Hill, 1994.

Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	radians/$(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022\times10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	$\propto$ or $m_{\propto}$ $\frac{1}{12}m\left(^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1}/N_A
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^{5} Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		–459.67°F (0°R)	–273.16°C (0 K)
Velocity[3]			
Earth escape		3.67×10^{4} ft/sec	1.12×10^{4} m/s
light (vacuum)	c, c_0	9.84×10^{8} ft/sec	2.9979 (3.00) $\times10^{8}$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.

Appendix C: Fundamental Constants

Table Note 1

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	∝B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/°R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	∝N		5.050×10^{-27} J/T
permeability of a vacuum	∝0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π	□		1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm-°R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02°F, 0.0888 psia	0.01109°C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol-°R 1.986 BTU/lbmol-°R	8314 J/kmol·K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of “lbmol” is an actual unit, not a misspelling.

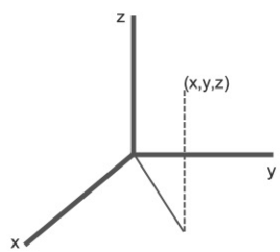
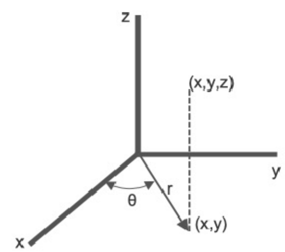
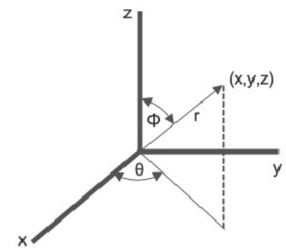
Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Appendix F: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\theta + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\phi + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\theta$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \theta & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\phi & \frac{1}{r}\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$